碩士學位 請求論文

# On the Quotient Structure of Column Finite Matrix Semiring

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## CONTENTS

# Ⅰ. INTRODUCTION

When A is a semiring and J is an ideal of A, the collection $\{x+J\}_{x\in A}$ of sets $x+J=\{x+j \mid j\in J\}$ need not be a partition of A.

P. J. Allen [1] defined Q−ideal and maximal homomorphism and establised Fundamental Theorem of Homomorphism in a large class of semirings.

Moreover, [3] builds the quotient structure in row finite matrix semirings.

This paper aims at proving an analogue of results for column finite matrix semirings as follows; if A is a semiring and J is a Q−ideal of A, then the collection $[A]_{CF}^{I\times I}$ of column finite matrices over A is a semiring, $[J]_{CF}^{I\times I}$ is a $[Q]_{CF}^{I\times I}$−ideal of $[A]_{CF}^{I\times I}$ and $[A]_{CF}^{I\times I}/[J]_{CF}^{I\times I}$ is isomorphic to $[A/J]_{CF}^{I\times I}$.

## II. PRELIMINARIES

**Definition 2.1** A non-empty set A together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$, respectively) will be called a *semiring* provided :

(1) addition is a commutative operation,

(2) there exists $o \in A$ such that $x+o=x$ and $x \cdot o=o \cdot x=o$ for all $x \in A$,

(3) multiplication distributes over addition both from the left and from the right.

**Definition 2.2** A non-empty subset J of a semiring A will be called an *ideal* if a, $b \in J$ and $r \in A$ implies $a+b \in J$, $ra \in J$ and $ar \in J$.



**Definition 2.3** A mapping $\phi$ from the semiring A into the semiring A′ will be called a *homomorphism* if $\phi(a+b)=\phi(a)+\phi(b)$ and $\phi(ab)=\phi(a)\phi(b)$ for each a, $b \in A$.

An isomorphism is an one-to-one homomorphism.

The semirings A and A′ will be called isomorphic (denoted by $A \cong A'$) if there exists an isomorphism from A onto A′.

**Definition 2.4** An ideal J in the semiring A will be called a *Q−ideal* if there exists a subset Q of A satisfying the following conditions;

(1) $\{q+J\}_{q\in Q}$ is a partition of A and

(2) if $q_1$, $q_2 \in Q$ such that $q_1 \neq q_2$, then $(q_1+J)\cap(q_2+J)=\phi$.

**Definition 2.5** A homomorphism $\phi$ from the semiring A onto the semiring A′ is said to be *maximal* if for each $a\in A'$ there exists $c_a \in \phi^{-1}(\{a\})$ such that $x+\ker\phi \subset c_a+\ker\phi$ for each $x\in\phi^{-1}(\{a\})$, where $\ker\phi=\{x\in A \mid \phi(x)=0\}$.

**Theorem 2.6** Let J be a Q−ideal in the semiring A. If $x\in A$, then there exists a unique $q\in Q$ such that $x+J\subset q+J$.

**Proof** : Let $x\in A$. Since $\{q+J\}_{q\in Q}$ is a partition of A, there exists $q\in Q$ such that $x\in q+J$.

If $y\in x+J$, there exists $i_1\in J$ such that $y=x+i_1$. Since $x\in q+J$, there exists $i_2\in J$ such that $x=q+i_2$.

Clearly, $y=x+i_1=(q+i_2)+i_1=q+(i_2+i_1)\in q+J$.

Thus $x+J\subset q+J$.

The uniqueness is an immediate result of part (2) of Definition 2.4.

Let J be a Q−ideal in the semiring A. In the view of the above result, we can define the binary operations $\oplus_Q$ and $\odot_Q$ on $\{q+J\}_{q\in Q}$ as follows;

(1) $(q_1+J)\oplus_Q(q_2+J)=q_3+J$ where $q_3$ is the unique element in Q such that $q_1+q_2+J\subset q_3+J$.

(2) $(q_1+J)\odot_Q(q_2+J)=q_3+J$ where $q_3$ is the unique element in Q such that $q_1q_2+J\subset q_3+J$.

The elements $q_1+J$ and $q_2+J$ in $\{q+J\}_{q\in Q}$ will be called equal (denoted by $q_1+J=q_2+J$) if and only if $q_1=q_2$.

**Theorem 2.7** If J is a Q−ideal in the semiring A, then

$A/J=(\{q+J\}_{q\in Q}, \oplus_Q, \odot_Q)$ is a semiring.

**Proof :** It is an easy matter to show that $\oplus_Q$ and $\odot_Q$ are associative operations, $\oplus_Q$ is a commutative operation, and $\odot_Q$ distributes over $\oplus_Q$ both from the left and from the right.

Define $\phi : A\rightarrow\{q+J\}_{q\in Q}$ by $\phi(x)=q+J$ where q is the unique element in Q such that $x+J\subset q+J$.

It can be shown that $\phi$ is a homomorphism from the semigroup $(A, +)$ onto the semigroup $(\{q+J\}_{q\in Q}, \oplus_Q)$ and $\phi$ is a homomorphism from the semigroup $(A, \cdot)$ onto the semigroup $(\{q+J\}_{q\in Q}, \odot_Q)$.



Since 0 is the identity in $(A, +)$, it follows that $\phi(0)=q^*+J$ is the identity in $(\{q+J\}_{q\in Q}, \oplus_Q)$.

Let $q\in Q$ and let $x\in A$ such that $\phi(x)=q+J$. Since $x\cdot 0=0\cdot x=0$, it is clear that

$q^*+J=\phi(0)=\phi(0\cdot x)=\phi(0)\cdot\phi(x)=(q^*+J)\odot_Q(q+J)$ and

$q^*+J=\phi(0)=\phi(x\cdot 0)=\phi(x)\phi(0)=(q+J)\odot_Q(q^*+J)$.

Thus, the element $q^*+J$ satisfies condition (2) in Definition 2.1.

**Theorem 2.8** Let J be an ideal in the semiring A. If $Q_1$ and $Q_2$ are subsets of A such that J is both a $Q_1$−ideal and a $Q_2$−ideal, then

$(\{q+J\}_{q\in Q_1}, \oplus_{Q_1}, \odot_{Q_1}) \cong (\{q+J\}_{q\in Q_2}, \oplus_{Q_2}, \odot_{Q_2})$.

**Proof :** Define $\phi : \{q+J\}_{q\in Q_1} \to \{q+J\}_{q\in Q_2}$ as follows ; If $q_1 \in Q_1$, then $\phi(q_1+J)=q_2+J$ where $q_2$ is the unique element in $Q_2$ such that $q_1+J \subset q_2+J$.

It can be shown that $\phi$ is an isomorphism from the semiring $(\{q+J\}_{q\in Q_1}, \oplus_{Q_1}, \odot_{Q_1})$ onto the semiring $(\{q+J\}_{q\in Q_2}, \oplus_{Q_2}, \odot_{Q_2})$.

If J is an ideal in the semiring A, then it is possible that J can be considered to be a Q−ideal with respect to many different subsets Q of A.

However, the preceding theorem implies that the structure $(\{q+J\}_{q\in Q}, \oplus_Q, \odot_Q)$ is "essentially independent" of the choice of Q.

Thus, if J is a Q−ideal in A, the semiring $(\{q+J\}_{q\in Q}, \oplus_Q, \odot_Q)$ will be denoted by A/J or (A/J, $\oplus$, $\odot$).



**Lemma 2.9** Let $\phi$ be a homomorphism from the semiring A onto the semiring A′.

If $\phi$ is maximal, then ker$\phi$ is a Q−ideal, where $Q=\{c_a\}_{a\in A'}$.

**Proof :** It is clear that $\bigcup_{a\in A'}(c_a+\ker\phi)=A$.

Let $c_a$ and $c_b$ be distinct elements in Q, i.e $a \neq b$

Assume $(c_a+\ker\phi) \cap (c_b+\ker\phi) \neq \phi$

then there exists k, $k' \in \ker\phi$ such that $c_a+k=c_b+k'$.

Thus $a=\phi(c_a)+\phi(k)=\phi(c_a+k)$

$=\phi(c_b+k')=\phi(c_b)+\phi(k')$

$=b$, which is a contradition.

It follows that $\ker\phi$ is a Q−ideal.

**Lemma 2.10** Let A, A', $\phi$ and Q be as stated in Lemma 2.9, and let $c_a$, $c_b$ and $c_c$ be elements in Q.

(1) If $c_a+c_b+\ker\phi \subset c_c+\ker\phi$, then $a+b=c$.

(2) If $c_a c_b+\ker\phi \subset c_c+\ker\phi$, then $ab=c$.

**Proof :** Since $c_a+c_b \in c_a+c_b+\ker\phi \subset c_c+\ker\phi$, there exists $k \in \ker\phi$ such that $c_a+c_b=c_c+k$.

Thus $a+b=\phi(c_a)+\phi(c_b)=\phi(c_a+c_b)$

$=\phi(c_c+k)=\phi(c_c)+\phi(k)=c$.



Since $c_a c_b \in c_a c_b+\ker\phi \subset c_c+\ker\phi$, there exists $k \in \ker\phi$ such that $c_a c_b=c_c+k$

Thus $ab=\phi(c_a)\phi(c_b)=\phi(c_a c_b)$

$=\phi(c_c+k)=\phi(c_c)+\phi(k)$

$=c$.

**Theorem 2.11** If $\phi$ is a maximal homomorphism from the semiring A onto the semiring A', then $A/\ker\phi \cong A'$.

**Proof :** Define $\Phi : A/\ker\phi \to A'$ by $\Phi(c_a+\ker\phi)=a$ for each $c_a \in Q$.

It is clear that $\Phi$ is an one-to-one function from $A/\ker\phi$ onto $A'$.

It will be shown that $\Phi$ is an isomorphism and the theorem will follow.

From the definition of addition in $A/\ker\phi$, it follows that

$\Phi[(c_a+\ker\phi)\oplus(c_b+\ker\phi)]=\Phi[c_c+\ker\phi]=c$ where $c_c$ is the unique element in Q such that $c_a+c_b+\ker\phi\subset c_c+\ker\phi$.

In the view of Lemma 2.10, it is clear that

$$\Phi(c_a+\ker\phi)+\Phi(c_b+\ker\phi)=a+b=c$$

$$=\Phi[(c_a+\ker\phi)\oplus(c_b+\ker\phi)].$$

The definition of multiplication in $A/\ker\phi$ implies

$$\Phi[(c_a+\ker\phi)\odot(c_b+\ker\phi)]=\Phi[c_c+\ker\phi]=c.$$

where $c_c$ is the unique element in Q such that $c_a c_b+\ker\phi\subset c_c+\ker\phi$.

In the view of Lemma 2.10, it is clear that

$$\Phi(c_a+\ker\phi)\ \Phi(c_b+\ker\phi)=ab=c$$

$$=\Phi[(c_a+\ker\phi)\odot(c_b+\ker\phi)].$$

# Ⅲ. THE QUOTIENT OF COLUMN FINITE MATRIX SEMIRING

Consider a semiring A and non-empty countable index set I. Mappings $M: I \times I \rightarrow A$ are called matrices over A. The values of M are denoted by $m_{ij}$, where $i, j \in I$. The values $m_{ij}$ are also referred to as the entries of the matrix. In particular, $m_{ij}$ is called the $(i, j)$-entry of M.

The matrix M is denoted by $[m_{ij}]$ and the collection of all matrices M over A as defined above is denoted by $[A]^{I \times I}$.

For each $M=[m_{ij}] \in [A]^{I \times I}$ and each $j \in I$, consider the set of indices $C(M, j)=\{i \in I \mid m_{ij} \neq 0\}$.

Then M is called a ***column finite matrix*** iff $C(M, j)$ is finite for all $j \in I$.

The collection of all column finite matrices over A as defined above is denoted by $[A]_{CF}^{I \times I}$.



**Theorem 3.1** If A is a semiring, then $[A]_{CF}^{I \times I}$ is a semiring.

**Proof** : For $M=[m_{ij}]$, $N=[n_{ij}] \in [A]_{CF}^{I \times I}$, we define the addition and the multiplication by

$$M+N = [m_{ij}+n_{ij}] \text{ for all } i, j \in I \text{ and}$$

$$MN = [\sum_{j \in I} m_{ij} n_{jk}] \text{ for all } i, k \in I.$$

Then the addition and multiplication are well-defined operations on $[A]_{CF}^{I\times I}$ as follows ;

For each $j \in I$, if $l \in C(M+N, j)$, then $m_{lj}+n_{lj} \neq 0$, i. e. $m_{lj} \neq 0$ or $n_{lj} \neq 0$.

Thus. $l \in C(M, j)$ or $l \in C(N, j)$, i. e. $l \in C(M, j) \cup (N, j)$

Hence $C(M+N, j) \subset C(M, j) \cup C(N, j)$ is finite for all $j \in I$.

For each $j \in I$, if $l \in C(MN, j)$, then $\sum_{k \in I} m_{lk} n_{kj} = \sum_{k \in C(N, j)} m_{lk} n_{kj} \neq 0$.

Thus there exists $k_0 \in C(N, j)$ such that $m_{lk_0} n_{k_0 j} \neq 0$.

Thus $m_{lk_0} \neq 0$ i. e. $l \in C(M, k_0)$.

Thus $l \in \bigcup_{k \in C(N, j)} C(M, k)$.

Hence $C(NN. j) \subset \bigcup_{k \in C(N, j)} C(M, k)$ is finite for all $j \in I$.

Now we introduce the zero matrix denoted by 0 that the entries of 0 are 0.

Then 0 is an additive zero.

Furthermore, the multiplication is associative.



For, let $L=[a_{ij}] \in [A]_{CF}^{I\times I}$, then

$$
\begin{aligned}
(LM)N &= \left(\sum_{k \in I} a_{ik}\, m_{kj}\right) N \\
&= \sum_{l \in I} \left[ \left( \sum_{k \in I} a_{ik} m_{kl} \right) n_{lj} \right] \\
&= \sum_{l \in I} \left[ \sum_{k \in I} \{ (a_{ik} m_{kl}) n_{lj} \} \right] \\
&= \sum_{l \in I} \left[ \sum_{k \in I} \{ a_{ik} (m_{kl}\, n_{lj}) \} \right] \\
&= \sum_{k \in I} \left[ \sum_{l \in I} \{ a_{ik} (m_{kl}\, n_{lj}) \} \right]
\end{aligned}
$$

$$= \sum_{k\in I} [\, a_{ik}( \sum_{l\in I} m_{kl}\, n_{lj})]$$

$$= L\,( \sum_{l\in I} m_{kl}\, n_{lj})$$

$$= L\,(MN).$$

It is clear that the addition is commutative, associative and distributes over addition both from the left and from the right.

Hence $[A]^{I\times I}_{CF}$ is also a semiring.

**Corollary 3.2** If A is a semiring and J is a Q−ideal of A, then $[A/J]^{I\times I}_{CF}$ is a semiring.

**Proof.** It is obvious by Theorerm 2.7 and Theorem 3.1. In this corollary, the binary operations are defined as follows;

(1) $[q'_{ij}+J]+[q''_{ij}+J]=[q_{ij}+J]$ where $q'_{ij}+q''_{ij}+J\subset q_{ij}+J$ for all $i, j\in I$.



(2) $[q'_{ij}+J][q''_{ij}+J]=[q_{ij}+J]$ where $\sum_{k\in I} q'_{ik}\, q''_{kj}+J\subset q_{ij}+J$ for all $i, j\in I$.

Since $N=[q''_{ij}+J]$ is column finite, the range of k in (2) is $C(N, j)$. So the range of k is finite.

**Theorem 3.3** If A is a semiring and J is a Q−ideal in A, then $[J]^{I\times I}_{CF}$ is a $[Q]^{I\times I}_{CF}$−ideal in $[A]^{I\times I}_{CF}$.

**Proof.** It is clear that $[J]^{I\times I}_{CF}$ is an ideal in $[A]^{I\times I}_{CF}$.

(1) Suppose $[m_{ij}]\in[A/J]^{I\times I}_{CF}$.

Since $m_{ij}\in A$ for all $i, j\in I$ and J is a Q−ideal in A, $m_{ij}\in \bigcup_{q\in Q}\{q+J\}$ for all

$i, j \in I$, i.e. for all $i, j \in I$, $m_{ij} = q_{ij} + n_{ij}$ for some $q_{ij} \in Q$ and some $n_{ij} \in J$, $[m_{ij}] = [q_{ij} + n_{ij}] = [q_{ij}] + [n_{ij}] \in P + [J]_{CF}^{I \times I}$ for some $P = [q_{ij}] \in [Q]_{CF}^{I \times I}$.

Hence $[m_{ij}] \in \bigcup_{P \in [Q]_{CF}^{I \times I}} \{P + [J]_{CF}^{I \times I}\}$.

(2) Let $[p_{ij}]$ and $[q_{ij}]$be in $[Q]_{CF}^{I \times I}$ and let $[p_{ij}] = [q_{ij}]$. Then there exist $i, j \in I$ such that $p_{ij} \neq q_{ij}$.

Since J is a Q−ideal in A, $(p_{ij} + J) \cap (q_{ij} + J) = \phi$. So $p_{ij} + m \neq q_{ij} + n$ for all $m, n \in J$.

Consequently, the $(i, j)$−entry of every matrices in $[p_{ij}] + [J]_{CF}^{I \times I}$ is different from the $(i, j)$−entry of every matrices in $[q_{ij}] + [J]_{CF}^{I \times I}$.

Thus $([p_{ij}] + [J]_{CF}^{I \times I}) \cap ([q_{ij}] + [J]_{CF}^{I \times I}) = \phi$.

Hence $[J]_{CF}^{I \times I}$ is a $[Q]_{CF}^{I \times I}$−ideal in $[A]_{CF}^{I \times I}$.

**Corollary 3.4** If A is a semiring and J is a Q−ideal in A, then

$[A]_{CF}^{I \times I} / [J]_{CF}^{I \times I} = (\{P + [J]_{CF}^{I \times I}\}_{P \in [Q]_{CF}^{I \times I}}, \oplus, \odot)$ is a semiring.

**Proof.** This corollary is the immediate result of Theorem 3.3 and Theorem 2.7.



The operation are as follows;

(1) $(P_1 + [J]_{CF}^{I \times I}) \oplus (P_2 + [J]_{CF}^{I \times I}) = P + [J]_{CF}^{I \times I}$ where $P_1 + P_2 + [J]_{CF}^{I \times I} \subset P + [J]_{CF}^{I \times I}$ and

(2) $(P_1 + [J]_{CF}^{I \times I}) \odot (P_2 + [J]_{CF}^{I \times I}) = P + [J]_{CF}^{I \times I}$ where $P_1 P_2 + [J]_{CF}^{I \times I} \subset P + [J]_{CF}^{I \times I}$.

**Proposition 3.5** If J is a Q−ideal in a semiring A, then J is a zero−element in A/J.

**Proof :** Let $q^* \in Q$ such that $J \subset q^* + J$. Then $q^* + J$ is a zero-element in

A/J by Theorem 2.7.

Since $0 \in J \subset q^* + J$, $0 = q^* + i$ for some $i \in J$.

Thus $q^* + J = q^* + 0 + J = q^* + q^* + i + J \subset q^* + q^* + J$.

Since $q^* + q^* + J$ is contained in a unique coset $q' + J$ where $q' \in Q$, $q' + J = q^* + J$ i.e. $q^* + q^* + J = q^* + J$.

Thus $q^* + q^* = q^* + i_1$ for some $i_1 \in J$.

Hence $q^* + J = q^* + 0 + J = q^* + q^* + i + J$

$= q^* + i_1 + i + J = q^* + i + i_1 + J$

$= 0 + i_1 + J \subset J$.

Therefore $q^* + J = J$.

**Proposition 3.6** A Q−ideal J of a semiring A is a k-ideal of A.

**Proof :** Recall that an ideal J is a k-ideal if $x + i \in J$, where $x \in A$ and $i \in J$, implies $x \in J$.



Suppose $x + i \in J$, where $x \in A$ and $i \in J$. Then there exists a unique coset $q + J$ such that $x + J \subset q + J$. Thus $x + i \in q + J$.

Since $x + i \in J = q^* + J$, $q = q^*$.

Therefore $x \in x + J \subset q + J = q^* + J = J$.

**Theorem 3.7** If A is a semiring and J is a Q−ideal in A, then $[A]_{CF}^{|X|} / [J]_{CF}^{|X|}$ is isomorphic to $[A/J]_{CF}^{|X|}$.

**Proof :** For each $m_{ij} \in A$, there exists a unique $q_{ij} \in Q$ such that

$m_{ij}+J \subset q_{ij}+J$ by Theorem 2.6.

Define the map $\phi : [A]_{CF}^{I\times I} \to [A/J]_{CF}^{I\times I}$ by $\phi([m_{ij}])=[q_{ij}+J]$ for each $[m_{ij}] \in [A]_{CF}^{I\times I}$, where $m_{ij}+J \subset q_{ij}+J$ for each $i, j \in I$.

Let $\phi([a_{ij}])=[q_{ij}+J]$, $\phi([b_{ij}])=[q'_{ij}+J]$ and $\phi\ [a_{ij}+b_{ij}]=[q''_{ij}+J]$.

Then $a_{ij}+J \subset q_{ij}+J$, $b_{ij}+J \subset q'_{ij}+J$, $a_{ij}+b_{ij}+J \subset q''_{ij}+J$ and also $a_{ij}+b_{ij}+J \subset q_{ij}+q'_{ij}+J$.

Thus $q_{ij}+q'_{ij}+J \subset q''_{ij}+J$.

To prove that $[q_{ij}+J]+[q'_{ij}+J]=[q''_{ij}+J]$,

let $[q_{ij}+J]+[q'_{ij}+J]=[q'''_{ij}+J]$, then $q_{ij}+q'_{ij}+J \subset q'''_{ij}+J$.

So, $q'''_{ij}+J=q''_{ij}+J$ for all $i, j \in I$, i.e. $[q'''_{ij}+J]=[q''_{ij}+J]$.

Thus $\phi\ ([a_{ij}]+[b_{ij}])=\phi([a_{ij}])+\phi([b_{ij}])$.

Similarly, we can show that $\phi([a_{ij}][b_{ij}])=\phi([a_{ij}])\phi([b_{ij}])$.

Hence $\phi$ is a homomorphism from $[A]_{CF}^{I\times I}$ onto $[A/J]_{CF}^{I\times I}$.



And $\ker\phi=[J]_{CF}^{I\times I}$ is clear by proposition 3.5 and proposition 3.6.

For each $(q_{ij}+J) \in [A/J]_{CF}^{I\times I}$, $[q_{ij}] \in \phi^{-1}([q_{ij}+J])$.

If $[a_{ij}] \in \phi^{-1}([q_{ij}+J])$, then $a_{ij}+J \subset q_{ij}+J$ for all $i, j \in I$.

Thus $[a_{ij}]+\ker\phi=[a_{ij}]+[J]_{CF}^{I\times I} \subset [q_{ij}]+[J]_{CF}^{I\times I}=[q_{ij}]+\ker\phi$.

Hence $\phi$ is a maximal homomorphism from the semiring $[A]_{CF}^{I\times I}$ onto the semiring $[A/J]_{CF}^{I\times I}$.

Therefore $[A]_{CF}^{I\times I}/[J]_{CF}^{I\times I}$ is isomorphic to $[A/J]_{CF}^{I\times I}$ by Theorem 2.11.

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**國文抄錄**

# 열유한 행렬 반환의 몫의 구조에 관하여

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이 논문에서는 A가 semiring이고 J가 Q−ideal이면, $[J]_{CF}^{I\times I}$ 는 $[A]_{CF}^{I\times I}$ 에서 $[Q]_{CF}^{I\times I}$ −ideal이 되어 $[A]_{CF}^{I\times I}/[J]_{CF}^{I\times I}$ 는 semiring이 됨을 보였고, 또 $[A]_{CF}^{I\times I}/[J]_{CF}^{I\times I}$ 와 $[A/J]_{CF}^{I\times I}$ 는 서로 동형임을 보였다.

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