IL NUOVO CIMENTO Vol. 112 B, N. 10 Ottobre 1997

The Bateman method for multichannel scattering theory (*)

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(ricevuto il 7 Gennaio 1997; approvato il 25 Marzo 1997)

Summary. — Accuracy and convergence of the Bateman method are investigated for calculating the transition amplitude in multichannel scattering theory. This approximation method is applied to the calculation of elastic amplitude. The calculated results are remarkably accurate compared with those of exactly solvable multichannel model.

PACS 03.65.Nk - Nonrelativistic scattering theory.

1. - Introduction

During the past two decades, there has been rapid development of the Schwinger variational method for studying scattering process, bound, and resonance state [1-5]. In this paper, we present a Bateman method [6] which is the special case of the Schwinger variational method [7-9], to calculate transition amplitudes in a multichannel scattering theory.

In sect. 2, we briefly describe the Bateman method. In sect. 3, we describe an exactly solvable model for multichannel scattering problem which will be used to access the accuracy of the Bateman method. The Bateman approximation method for multichannel scattering problem is described in sect. 4. Application of the Bateman method is described and comparisons of results are presented in sect. 5. Summary and conclusions are given in sect. 6.

2. - Bateman method

We consider the formal identity for the potential V,

$$V = VV^{-1}V = \sum_{i,j} V|i\rangle\langle i|V^{-1}|j\rangle\langle j|V\,, \tag{1}$$

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where $|i\rangle$ and $|j\rangle$ are, in general, different complete sets. If we truncate the summation over the complete sets in eq. (1), we obtain the separable approximation

$$V^{(M)} = \sum_{i,j=0}^{M} V|\eta_i\rangle d_{ij}^{-1}\langle\chi_j|V, \tag{2}$$

where $d_{ij} = \langle\chi_i|V|\eta_j\rangle$. Equation (2) represents an interpolation process since $V^{(M)}|\eta_i\rangle = V|\eta_i\rangle$ and $\langle\chi_i|V^{(M)} = \langle\chi_i|V$. One can show [9] that the well-known Bubnov-Galerkin, Hilbert-Schmidt and Bateman methods are special cases of the expression eq. (2) for appropriate choice of the functions $|\eta\rangle$ and $|\chi\rangle$. The separable approximation equation (2) is closely related to Weinstein's method in the formulation of Bazley and Fox [10].

In the case of multichannel theory, the total Hamiltonian can be written as $H = H_0 + V$, where $\langle n|H_0|m\rangle$ is a diagonal matrix and $\langle n|V|m\rangle$ is a non-diagonal matrix, and $|n\rangle$ and $|m\rangle$ are channel (basis) functions. The Bateman expression for $\langle n|V|m\rangle$ can be written as [6]

$$\langle n|V|m\rangle = \sum_{i,j=0}^{M} \langle n|V|\alpha_i\rangle\, d_{ij}^{-1}\langle\alpha_j|V|m\rangle, \tag{3}$$

where, $d_{ij} = \langle\alpha_i|V|\alpha_j\rangle$ and $n, m = 0, 1, 2, \ldots$. The main problem is how to choose points $\alpha_0, \alpha_1, \alpha_2 \ldots, \alpha_M$, when M is not large. When $\langle n|V|m\rangle$ is analytical function of n and m, we propose to use a condition, $\frac{\partial|f|^2}{\partial\alpha_i} = 0$, to fix $\alpha_0, \alpha_1, \alpha_2, \ldots \alpha_M$ (f is the amplitude).

3. – Exactly solvable model for multichannel scattering problem

The Schrödinger equation for a multichannel system can be written as

$$-\frac{\hbar^2}{2\mu}\nabla^2\Psi_\alpha(\mathbf{r}) + \sum_\beta V_{\alpha\beta}(\mathbf{r}, \mathbf{r}')\, C_\beta(\mathbf{r}') = (E - \varepsilon_\alpha)\,\Psi_\alpha(\mathbf{r}), \tag{4}$$

where μ is the reduced mass and ε_α the excitation energy in α-th channel. The solution of eq. (4) can be expressed in terms of the integral

$$\Psi_\alpha(\mathbf{r}) = \phi_\alpha(\mathbf{r})\,\delta_{\alpha 0} - \frac{\mu}{2\pi\hbar^2}\int \frac{e^{ik_\alpha|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\sum_\beta V_{\alpha\beta}(\mathbf{r}', \mathbf{r}'')\Psi_\beta(\mathbf{r}'')\,d\mathbf{r}'\,d\mathbf{r}'', \tag{5}$$

where $\phi_\alpha(\mathbf{r}) = e^{i\mathbf{k}_\alpha\cdot\mathbf{r}}$, $k_\alpha = \frac{1}{\hbar}\sqrt{2\mu(E - \varepsilon_\alpha)}$, and $V_{\alpha\beta}(\mathbf{r}, \mathbf{r}') = \langle\mathbf{r}|V_{\alpha\beta}|\mathbf{r}'\rangle$. For large r ($r \to \infty$), eq. (5) can be written as

$$\Psi_\alpha(\mathbf{r}) \to e^{i\mathbf{k}_\alpha\cdot\mathbf{r}}\,\delta_{\alpha 0} + f_{0\to\alpha}\,\frac{e^{ik_\alpha r}}{r}, \tag{6}$$

where $f_{0\to\alpha}$ is the scattering amplitude given by

$$f_{0\to\alpha} = -\frac{\mu}{2\pi\hbar^2}\sum_\beta\langle\phi_\alpha|V_{\alpha\beta}|\Psi_\beta\rangle. \tag{7}$$

We can rewrite eq. (5) as

$$|\Psi_\alpha\rangle = |\phi_\alpha\rangle\,\delta_{\alpha 0} + \frac{1}{E-\varepsilon_\alpha - H_0 + i\varepsilon}\sum_\beta V_{\alpha\beta}\,|\Psi_\beta\rangle. \tag{8}$$

If we assume $V_{\alpha\beta}$ as

$$V_{\alpha\beta}(\mathbf{r},\mathbf{r}') = \langle \mathbf{r}|V_{\alpha\beta}|\mathbf{r}'\rangle = |u_\alpha(\mathbf{r})\rangle\, g_{\alpha\beta}\langle u_\beta(\mathbf{r}')|\ , \tag{9}$$

eq. (8) becomes

$$|\Psi_\alpha\rangle = |\phi_\alpha\rangle\,\delta_{\alpha 0} + \frac{1}{E-\varepsilon_\alpha - H_0 + i\varepsilon}\,|u_\alpha\rangle\, C_\alpha, \tag{10}$$

where $g_{\alpha\beta}$ are the channel-coupling strengths, $u_\alpha(\mathbf{r}) = \langle \mathbf{r}|u_\alpha\rangle$ and $u_\beta(\mathbf{r}') = \langle \mathbf{r}'|u_\beta\rangle$ are scalar functions of the relative (projectile-target) coordinate, and C_α is given by

$$C_\alpha = \sum_\beta g_{\alpha\beta}\langle u_\beta|\Psi_\beta\rangle, \tag{11}$$

which satisfies the following relation:

$$C_\alpha = \sum_\beta g_{\alpha\beta}\langle u_\beta|\phi_\beta\rangle\delta_{\beta 0} + \sum_\beta g_{\alpha\beta}\langle u_\beta|\frac{1}{E-\varepsilon_\beta - H_0 + i\varepsilon}|u_\beta\rangle\, C_\beta\,. \tag{12}$$

If we substitute

$$G_\beta = \langle u_\beta|\frac{1}{E-\varepsilon_\beta - H_0 + i\varepsilon}|u_\beta\rangle, \tag{13}$$

and

$$C_\alpha = \langle u_0|\phi_0\rangle f_\alpha(E), \tag{14}$$

into eq. (12), we can obtain for $f_\alpha(E)$ the following equation:

$$\sum_\beta[\delta_{\alpha\beta} - g_{\alpha\beta}G_\beta]\cdot f_\beta(E) = g_{\alpha 0}\,. \tag{15}$$

Using eqs. (7), (9), (11) and (14), we can express the scattering amplitude $f_{0\to\alpha}$ as

$$f_{0\to\alpha} = -\frac{\mu}{2\pi\hbar^2}\langle\phi_\alpha|u_\alpha\rangle\langle u_0|\phi_0\rangle f_\alpha(E). \tag{16}$$

If we define transition amplitude $T_{0\to\alpha}$, by renormalizing the scattering amplitude $f_{0\to\alpha}$ as

$$T_{0\to\alpha} = -\frac{2\pi\hbar^2}{\mu}f_{0\to\alpha}\,, \tag{17}$$

the transition amplitude can be written as

$$T_{0\to\alpha} = \langle\phi_\alpha|u_\alpha\rangle\langle u_0|\phi_0\rangle f_\alpha(E), \tag{18}$$

where $\langle \phi_\alpha | u_\alpha \rangle$ is given by

$$(19) \qquad \langle \phi_\alpha | u_\alpha \rangle = \int \phi_\alpha(\mathbf{r})\, u_\alpha(\mathbf{r})\, d\mathbf{r} = \frac{1}{(2\pi)^3} \int \phi_\alpha(\mathbf{k})\, u_\alpha(\mathbf{k})\, d\mathbf{k}\,.$$

Using the following relation:

$$(20) \qquad \phi_\alpha(\mathbf{k}) = \int e^{i(\mathbf{k}-\mathbf{k}_\alpha)\cdot\mathbf{r}}\, d\mathbf{r} = (2\pi)^3 \delta(\mathbf{k}-\mathbf{k}_\alpha)\,,$$

eq. (19) can be written as

$$(21) \qquad \langle \phi_\alpha | u_\alpha \rangle = u_\alpha(\mathbf{k}_\alpha) = \langle \mathbf{k}_\alpha | u_\alpha \rangle\,.$$

Therefore, the transition amplitude, eq. (18), can be expressed as

$$(22) \qquad T_{0\to\alpha} = \langle \mathbf{k}_\alpha | u_\alpha \rangle \langle u_0 | \mathbf{k}_0 \rangle \cdot f_\alpha(E)\,,$$

where $f_\alpha(E)$ can be calculated from eq. (13) and eq. (15). The coefficient G_β required for calculating $f_\alpha(E)$ is given by

$$(23) \qquad G_\beta = \frac{1}{(2\pi)^3} \int \frac{(u_\beta(\mathbf{k}))^2 k^2\, dk\, d\Omega}{E - \varepsilon_\beta - \dfrac{\hbar^2 k^2}{2\mu} + i\varepsilon}\,.$$

For the function $u_\beta(\mathbf{k})$, we will take the Yamaguchi form factor [11]

$$(24) \qquad u_\beta(\mathbf{k}) = \frac{1}{k^2 + \gamma_\beta^2}\,,$$

which leads to

$$(25) \qquad G_\beta = -\frac{\mu}{4\pi\hbar^2} \frac{1}{\gamma_\beta(\gamma_\beta - ik_\beta)^2}\,.$$

For the coupling strength, we take the expression given by

$$(26) \qquad g_{\alpha\beta} = -\delta \exp\left[-\left(\frac{\alpha-\beta}{a}\right)^2\right] = -\delta \tilde{g}_{\alpha\beta}\,,$$

where the parameter a measures the extent to which the strength of the coupling is distributed among distant channels. By inserting eq. (26) into eq. (15), eq. (15) becomes

$$(27) \qquad \sum_\beta [\delta_{\alpha\beta} + \tilde{g}_{\alpha\beta}\tilde{G}_\beta] \cdot f_\beta(E) = \tilde{g}_{\alpha 0}\,,$$

where

$$(28) \qquad \tilde{G}_\beta = -\left[\frac{\gamma}{\Delta - i\sqrt{E_\beta}}\right]^2$$

and

$$f_{\beta}(E) = -\frac{f_\beta(E)}{\delta}\,. \tag{29}$$

In eq. (28), γ and Δ are parameters given by

$$\Delta = \frac{\hbar}{\sqrt{2\mu}}\gamma_\beta\,, \qquad \gamma = \sqrt{\frac{\delta}{8\pi\gamma_\beta}}\,. \tag{30}$$

In this study, we will treat γ_β as constant, as Breitschaft *et al.* [12].

4. – Bateman approximation for multichannel scattering problem

As an approximated method to solve $f_\alpha(E)$ in eqs. (27) and (29), we use the Bateman approximation to $\tilde{g}_{\alpha\beta}$,

$$\tilde{g}_{\alpha\beta} \rightarrow \tilde{g}^{(M)}_{\alpha\beta} = \sum_{i,j=0}^{M} \tilde{g}_{\alpha l_i} d_{ij}^{-1} \tilde{g}_{l_j\beta}\,, \tag{31}$$

where $M < N-1$ (N is the number of channel), $d_{ij} = \tilde{g}_{l_i l_j}$. And $l_0 = 0$ and $l_1, l_2, \ldots, l_M$ are parameters to be determined. Inserting eq. (31) into eq. (27), we obtain the following relation:

$$\tilde{f}^{(M)}_\alpha(E) = \tilde{g}_{\alpha 0} - \sum_{i=0}^{M} \tilde{g}_{\alpha l_i} S_i\,, \tag{32}$$

where

$$S_i = \sum_\beta \sum_j d_{ij}^{-1} \tilde{g}_{l_j\beta} \tilde{G}_\beta \tilde{f}^{(M)}_\beta(E) \tag{33}$$

which satisfies the following relation:

$$\sum_{i=0}^{M}\left[\tilde{g}_{l_j l_i} + \sum_\beta \tilde{g}_{l_j\beta}\tilde{G}_\beta \tilde{g}_{\beta l_i}\right] S_i = \sum_\beta \tilde{g}_{l_j\beta}\tilde{G}_\beta \tilde{g}_{\beta 0}\,. \tag{34}$$

In the case of $M=0$, we can rewrite

$$S_0 = \frac{B}{A}\,, \tag{35}$$

where

$$A = \tilde{g}_{00} + \sum_{\beta=0}^{N-1} \tilde{g}_{0\beta}\tilde{G}_\beta \tilde{g}_{\beta 0}\,, \tag{36}$$

and

$$B = \sum_{\beta=0}^{N-1} \tilde{g}_{0\beta}\tilde{G}_\beta \tilde{g}_{\beta 0}\,. \tag{37}$$

We can calculate $\tilde{f}_{\alpha}^{(0)}$ by inserting eq. (35) into eq. (32). The transition amplitude $T_{0\to\alpha}^{(0)}$ can be calculated by inserting $f_{\alpha}^{(0)}(E)(=-\delta\tilde{f}_{\alpha}^{(0)}(E))$ into eq. (22). For the $M=1$ case, S_i, *i.e.* S_0 and S_1 are found by solving eq. (34), therefore, $\tilde{f}_{\alpha}^{(1)}(E)$ are obtained by using eq. (32). By inserting $f_{\alpha}^{(1)}(E)$ $(=-\delta\tilde{f}_{\alpha}^{(1)}(E))$ into eq. (22), the transition amplitude $T_{0\to\alpha}^{(1)}$ can be calculated.

5. - Applications and comparisons

As in the preceding sections, we have presented Bateman approximation method to calculate the transition amplitude in multichannel model. Using this approximation, we have obtained the simple form for $f_{\alpha}(E)$ related to the transition amplitude. We have applied the Bateman approximation method to the calculation of elastic transition amplitude ($T_{0\to0}$). For the comparison, we have also calculated the corresponding transition amplitudes by using both exactly solvable multichannel model and Born approximation. Four parameters (Δ, γ , E and a) are needed in order to calculate transition amplitude for exactly solvable multichannel model ($T_{0\to0}^{(ex)}$) and for the $M=0$ case of Bateman approximation ($T_{0\to0}^{(0)}$). In the case of calculation for transition amplitude by using Bateman approximation ($T_{0\to0}^{(1)}$) for the $M=1$ case, l_1 values should be added to the above four parameters and this parameter is determined by the

TABLE I. - *Calculated results of real and imaginary parts of exactly solvable model* ($T_{0\to0}^{(ex)}$), *Bateman approximation* ($T_{0\to0}^{(M)}$ *for* $M=0$ *and* 1) *and Born approximation* ($T_{0\to0}^{(Born)}$) *for elastic transition amplitude. The number of channel is* 10.

E	a/l_1	$\mathrm{Re}(T_{0\to0}^{(ex)})$/ $\mathrm{Im}(T_{0\to0}^{(ex)})$	$\mathrm{Re}(T_{0\to0}^{(0)})$/ $\mathrm{Im}(T_{0\to0}^{(0)})$	$\mathrm{Re}(T_{0\to0}^{(1)})$/ $\mathrm{Im}(T_{0\to0}^{(1)})$	$T_{0\to0}^{(Born)}$
20.0	20.0	-3.64392×10^{-3}	-3.28912×10^{-3}	-3.64388×10^{-3}	-9.73092×10^{-3}
	6	-8.90100×10^{-5}	-9.84224×10^{-5}	-8.90120×10^{-5}	
20.0	35.0	-3.23887×10^{-3}	-3.09725×10^{-3}	-3.23887×10^{-3}	-9.73092×10^{-3}
	6	-9.15909×10^{-5}	-9.55015×10^{-5}	-9.15909×10^{-5}	
20.0	50.0	-3.12126×10^{-3}	-3.04820×10^{-3}	-3.12126×10^{-3}	-9.73092×10^{-3}
	6	-9.26564×10^{-5}	-9.46992×10^{-5}	-9.26565×10^{-5}	
60.0	20.0	-6.73015×10^{-4}	-6.58269×10^{-4}	-6.73013×10^{-4}	-1.08193×10^{-3}
	6	-6.23148×10^{-6}	-6.67876×10^{-6}	-6.23152×10^{-6}	
60.0	35.0	-6.41088×10^{-4}	-6.35223×10^{-4}	-6.41088×10^{-4}	-1.08193×10^{-3}
	6	-6.62051×10^{-6}	-6.79687×10^{-6}	-6.62052×10^{-6}	
60.0	50.0	-6.32169×10^{-4}	-6.29149×10^{-4}	-6.32169×10^{-4}	-1.08193×10^{-3}
	6	-6.73312×10^{-6}	-6.82376×10^{-6}	-6.73313×10^{-6}	
100.0	20.0	-2.83969×10^{-4}	-2.81241×10^{-4}	-2.83969×10^{-4}	-3.89548×10^{-4}
	6	-1.49004×10^{-6}	-1.56718×10^{-6}	-1.49005×10^{-6}	
100.0	35.0	-2.75283×10^{-4}	-2.74192×10^{-4}	-2.75283×10^{-4}	-3.89548×10^{-4}
	6	-1.59695×10^{-6}	-1.62753×10^{-6}	-1.59695×10^{-6}	
100.0	50.0	-2.72873×10^{-4}	-2.72309×10^{-4}	-2.72873×10^{-4}	-3.89548×10^{-4}
	6	-1.62704×10^{-6}	-1.64278×10^{-6}	-1.62704×10^{-6}	

TABLE II. – *Calculated results of real and imaginary parts of exactly solvable model* ($T^{(ex)}_{0\to 0}$), *Bateman approximation* ($T^{(M)}_{0\to 0}$ *for* $M = 0$ **and** 1) *and Born approximation* ($T^{(Born)}_{0\to 0}$) *for elastic transition amplitude. The number of channel is* 50.

E	a/l_1	$\mathrm{Re}(T^{(ex)}_{0\to 0})$/ $\mathrm{Im}(T^{(ex)}_{0\to 0})$	$\mathrm{Re}(T^{(0)}_{0\to 0})$/ $\mathrm{Im}(T^{(0)}_{0\to 0})$	$\mathrm{Re}(T^{(1)}_{0\to 0})$/ $\mathrm{Im}(T^{(1)}_{0\to 0})$	$T^{(Born)}_{0\to 0}$
20.0	20.0	-3.44832×10^{-3}	-2.44840×10^{-3}	-3.42753×10^{-3}	-9.73092×10^{-3}
	5	-7.92161×10^{-5}	-8.36509×10^{-5}	-8.09301×10^{-5}	
20.0	35.0	-2.58618×10^{-3}	-1.55274×10^{-3}	-2.56136×10^{-3}	-9.73092×10^{-3}
	4	-6.60354×10^{-5}	-6.06174×10^{-5}	-6.77477×10^{-5}	
20.0	50.0	-2.13334×10^{-3}	-1.16738×10^{-3}	-2.12978×10^{-3}	-9.73092×10^{-3}
	8	-5.82355×10^{-5}	-4.83510×10^{-5}	-5.87656×10^{-5}	
60.0	20.0	-6.22221×10^{-4}	-5.50678×10^{-4}	-6.21971×10^{-4}	-1.08193×10^{-3}
	9	-5.74412×10^{-6}	-7.02731×10^{-6}	-5.74637×10^{-6}	
60.0	35.0	-5.04252×10^{-4}	-4.05255×10^{-4}	-5.04197×10^{-4}	-1.08193×10^{-3}
	14	-5.63032×10^{-6}	-6.62088×10^{-6}	-5.63370×10^{-6}	
60.0	50.0	-4.33010×10^{-4}	-3.29302×10^{-4}	-4.32960×10^{-4}	-1.08193×10^{-3}
	21	-5.23299×10^{-6}	-6.00652×10^{-6}	-5.23442×10^{-6}	
100.0	20.0	-2.63854×10^{-4}	-2.47202×10^{-4}	-2.63819×10^{-4}	-3.89548×10^{-4}
	11	-1.48346×10^{-6}	-1.81366×10^{-6}	-1.48336×10^{-6}	
100.0	35.0	-2.22419×10^{-4}	-1.95505×10^{-4}	-2.22391×10^{-4}	-3.89548×10^{-4}
	18	-1.59015×10^{-6}	-1.96114×10^{-6}	-1.59070×10^{-6}	
100.0	50.0	-1.95048×10^{-4}	-1.65456×10^{-4}	-1.95014×10^{-4}	-3.89548×10^{-4}
	24	-1.56966×10^{-6}	-1.92096×10^{-6}	-1.56985×10^{-6}	

following condition:

$$\frac{\partial |T^{(1)}_{0\to 0}|^2}{\partial l_1} = 0\,, \tag{38}$$

where

$$T^{(1)}_{0\to 0} = \mathrm{Re}(T^{(1)}_{0\to 0}) + i\,\mathrm{Im}(T^{(1)}_{0\to 0})\,. \tag{39}$$

For a case of $E = 20.0$, $a = 50.0$ and channel number = 90, the extremum point does not exist for $l_1 > 0$. Thus, we choose l_1 value for which the derivative of $|T^{(1)}_{0\to 0}|^2$ is minimum for $l_1 > 0$. The parameters used in this calculations are taken from Breitschaft *et al.* [12]: $\Delta = 0.1$, $\gamma = 2.094$ and equally spaced spectrum given by $\varepsilon_\alpha = 0.1\alpha$. In this schematic calculation, we choose units such that $c = \hbar = \mu = 1$. As examples of applications, three kinds of collision energies E (20.0, 60.0 and 100.0) are taken together with three different a values which are contained in coupling strength $\tilde{g}_{\alpha\beta}$ in eq. (26).

The calculations of the elastic transition amplitude ($T_{0\to 0}$) for three numbers of channel of 10, 50 and 90, and three a values of 20, 35 and 50, respectively are performed for three different energies. The calculated results are shown in tables I-III. For each channel, each table displays also calculated results of exactly solvable multichannel model ($T^{(ex)}_{0\to 0}$), the Bateman approximation ($T^{(M)}_{0\to 0}$ for $M=0$ and $M=1$) and the Born approximation ($T^{(Born)}_{0\to 0}$). In each table, we can find that our Bateman approximation for $M=0$ case gives reasonably good agreements with the results of exactly solvable

TABLE III. – *Calculated results of real and imaginary parts of exactly solvable model* ($T^{(ex)}_{0\to 0}$), *Bateman approximation* ($T^{(M)}_{0\to 0}$ *for* $M=0$ *and* 1) *and Born approximation* ($T^{(Born)}_{0\to 0}$) *for elastic transition amplitude. The number of channel is* 90.

E	a/l_1	$\mathrm{Re}(T^{(ex)}_{0\to 0})$/ $\mathrm{Im}(T^{(ex)}_{0\to 0})$	$\mathrm{Re}(T^{(0)}_{0\to 0})$/ $\mathrm{Im}(T^{(0)}_{0\to 0})$	$\mathrm{Re}(T^{(1)}_{0\to 0})$/ $\mathrm{Im}(T^{(1)}_{0\to 0})$	$T^{(Born)}_{0\to 0}$
20.0	20.0	-3.44825×10^{-3}	-2.44840×10^{-3}	-3.42750×10^{-3}	-9.73092×10^{-3}
	5	-7.92179×10^{-5}	-8.36509×10^{-5}	-8.09316×10^{-5}	
20.0	35.0	-2.58266×10^{-3}	-1.54510×10^{-3}	-2.54031×10^{-3}	-9.73092×10^{-3}
	3	-6.61188×10^{-5}	-6.04220×10^{-5}	-6.80287×10^{-5}	
20.0	50.0	-2.10501×10^{-3}	-1.10608×10^{-3}	-2.04474×10^{-3}	-9.73092×10^{-3}
	1	-5.72373×10^{-5}	-4.65069×10^{-5}	-5.98817×10^{-5}	
60.0	20.0	-6.22214×10^{-4}	-5.50678×10^{-4}	-6.21968×10^{-4}	-1.08193×10^{-3}
	9	-5.74448×10^{-6}	-7.02731×10^{-6}	-5.74651×10^{-6}	
60.0	35.0	-5.03457×10^{-4}	-4.04009×10^{-4}	-5.02559×10^{-4}	-1.08193×10^{-3}
	12	-5.62551×10^{-6}	-6.61387×10^{-6}	-5.66008×10^{-6}	
60.0	50.0	-4.29882×10^{-4}	-3.17680×10^{-4}	-4.28420×10^{-4}	-1.08193×10^{-3}
	14	-5.30098×10^{-6}	-5.89488×10^{-6}	-5.37675×10^{-6}	
100.0	20.0	-2.63854×10^{-4}	-2.47202×10^{-4}	-2.63818×10^{-4}	-3.89548×10^{-4}
	11	-1.48350×10^{-6}	-1.81366×10^{-6}	-1.48340×10^{-6}	
100.0	35.0	-2.22179×10^{-4}	-1.95040×10^{-4}	-2.22086×10^{-4}	-3.89548×10^{-4}
	16	-1.59439×10^{-6}	-1.96137×10^{-6}	-1.59836×10^{-6}	
100.0	50.0	-1.94466×10^{-4}	-1.60738×10^{-4}	-1.94340×10^{-4}	-3.89548×10^{-4}
	20	-1.58728×10^{-6}	-1.90746×10^{-6}	-1.59573×10^{-6}	

multichannel model. Furthermore, the results of $M=1$ case Bateman approximation agree remarkably well with results of exactly solvable multichannel model, showing substantial improvements with respect to the results of Bateman approximation for the $M=0$ case. As expected, the calculated transition amplitudes in multichannel model become much closer to those of Born approximation as the energy increases.

It is seen that the real parts of transition amplitude are much larger than imaginary parts, thus, these play an important role in the absolute value of transition amplitude. For a fixed number of channels and values of a , both the real and imaginary parts of the elastic transition amplitude decrease as the energy increases. Our calculated results with fixed values of energy and a depend on the number of channels used. For a fixed number of channels, the calculated results for the transition amplitude depend also on values of a and E.

6. – Summary and conclusions

In this paper, we have presented the Bateman approximation method to calculate the transition amplitude in multichannel scattering theory. This method produces accurate results compared with those of exactly solvable multichannel model. Particularly, the agreements between the results of $M=1$ case Bateman approximation and results of exactly solvable multichannel model are remarkably good. Thus, we conclude that our Bateman approximation method for the $M=1$ case (the parameter

l_1 is determined from eq. (38)) is a practical method for the calculation of the elastic amplitude in multichannel scattering model.

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One of the authors (YJK) acknowledges a partial support provided by a faculty exchange agreement between Purdue University and Cheju National University.

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