Hot-electron magnetophonon resonances in quasi-one-dimensional quantum structures

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Ⅰ. Introduction

Recent progress in nanofabrication technique has made it possible to observe the magnetophonon resonance (MPR) effect in semiconductors with low-dimensional structures and stimulated experimental and theoretical investigations[1-9] for the MPR effect which provides useful information on the transport properties of semiconductors, such as carrier relaxation mechanism, damping of the oscillations due to the electron-phonon interaction, the phonon frequencies, and band structure (i.e., the effective mass m^*). Therefore, a lot of work with respect to the ordinary(linear) and hot-electron MPR effects have been made[2-9] on these low-dimensional systems including two-dimensional electron-gas(2DEG), quasi-two-dimensional electron-gas(Q2DEG) and quasi-one-dimensional electron-gas(Q1DEG) systems. However, concerning the hot-electron MPR in Q1D quantum structures, to the best of our knowledge, we are not aware of experimental and theoretical work and are still at an initial stage both experimentally and theoretically. It is therefore needed to

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develop a theory which could analyze hot-electron MPR effects in Q1D quantum structures.

In this paper, we develop a thoery of hot-electron MPR in Q1D quantum structures, starting from the field-dependent conductivity formula[10] defined in the Ohm's law form of the nonlinear current density and study the physical characteristics of the hot-electron MPR effects in such structures. The origin of this formalism[10] dates back to the discovery of the theory of nonlinear static conductivity. Here we apply the theory to a simple model for a Q1DEG confined in the quantum structure subjected to the crossed electric and magnetic fields. We assume that the interaction with nonpolar optical phonons is the dominant scattering mechanism. Based on the model, we will evaluate the field-dependent transverse magnetoconductivity and the field-induced relaxation rate which is closely related to the electric-field-induced MPR (EFIMPR) effects.

The present paper is organized as follows: In Sec.II, we will describe the simple model of the system. In Sec.III, we present the field-dependent dc conductivity $\sigma_{xx}(E)$ formula given in the Ohm's law form of nonlinear electric current and the field-induced relaxation rate due to the collision process on the basis of nonlinear response theory[10] obtained previously. In Sec. IV, the field-induced relaxation rate for bulk optical phonon scatering in the Q1D quantum structure is calculated. The EFIMPR effect is also discussed for such a system, where the special attention is given to the unusual behavior of the EFIMPR line shape such as reduction in EFIMPR amplitude, conversion of EFIMPR maxima into minima or splitting of the EFIMPR peaks, and shift of EFIMPR peaks. Concluding remarks are given in Sec.V. In the Appendix, the explicit expression for the EFIMPR broadening parameter is derived for nonpolar optical phonon scattering.

II. Model for a Q1D quantum structure

We consider the high-field transport of electron gas in a Q1D quantum structure, where a static magnetic field $\vec{B}(\parallel \hat{z})$ and a dc electric field $\vec{E}(\parallel \hat{x})$ are applied perpendicularly to the barriers of the potential well (such as realized in the heterointerface) and along the lateral direction of their wells. The Q1D quantum structure is modeled by a parabolic potential well with the confinement frequency Ω

in the z direction, in order to see the effect of the confinement frequency in the nonlinear magnetoconductivity. Applying the effective mass approximation for conduction electrons confined in the Q1D quantum structure and taking the z abscissa origin at one interface, the one-particle Hamiltonian (h_{eE}) for such electrons subject to the crossed electric ($\vec{E}$) and magnetic ($\vec{B}$) fields, its normalized eigenfunctions ($|\lambda\rangle$) and eigenvalues (E_λ), in the Landau gauge $\vec{A}=(0, Bx, 0)$, are respectively, given as

$$h_{eE}=[\vec{p}+e\vec{A}]^2/2m^*+m^*\Omega^2z^2/2+eEx \tag{1}$$

$$|\lambda>=|N,n,k_y>=(1/L_y)^{1/2}\phi_N(x-x_\lambda)\phi_n(z)\exp(ik_y\, y) \tag{2}$$

$$E_\lambda=E_{nl}(k_y)=(n+1/2)\hbar\omega_c+(l+1/2)\hbar\Omega-eEx_\lambda+m^*V^{2}{}'/2 \tag{3}$$

where $\vec{p}$ is the momentum operator of a conduction electron, $N(=0,1,2,\cdots)$ and $n(=1,2,3,\cdots)$ are, respectively, the Landau- and subband-level indices, and $\Phi_n(x-x_\lambda)$ represents harmonic-oscillator wave functions, centered at $x=x_\lambda=-\tilde{b}\tilde{l}_B^2(k_y+m^*V_d\hbar)$. Here $\tilde{b}=\omega_c\tilde{\omega}_c$, k_y is the wave vector in the y direction, $V_d(=E/B)$ is the center drift velocity of the electron, and $\tilde{l}_B=(\hbar/m^*\tilde{\omega}_c)^{1/2}$ is the effective radius of the ground-state electron orbit in the (x, y) plane. $\tilde{\omega}_c=(\omega_c^2+\Omega^2)^{1/2}$ and $\tilde{m}=m^*\tilde{\omega}_c^2/\Omega^2$ means the renormalized cyclotron frequency with respect to the cyclotron frequency $\omega_c=eB/m^*$ and the renormalized mass with respect to the effective mass m^*, respectively. As shown in Eqs.(2) and (3), the electron energy spectrum in Q1D quantum wire is hybrid-quantized by the confinements in the x and z direction and the anisotropy in the single-electron energy spectrum in y and z direction is through the difference between m^* and $\tilde{m}$, which is mainly due to the confinement frequency. Furthermore, we see that the inclusion of the electric field effect is to shift the center position of the orbits by $E\omega_c/B\tilde{\omega}_c^2$ and to lift the k_y degeneracy of the energy spectrum. We shall designate a set of quantum numbers (N,n,k_y) by a greek letter lambda. lambda ±1 will then indicate the state

$(N\pm1,n,k_y)$. The dimensions of the sample are assumed to be $V=L_xL_yL_z$. It is interesting to note that Eqs.(2) and (3) enable us to see the dimensional crossover by simply varying the confining potential parameter, i.e., $\Omega\to 0$, which gives the Vasilopoulos et al.'s[3] and Suzuki's [8] result obtained for the Q2D quantum structure. The dependence of the single-electron energy spectrum in Eq.(3) on the width(L_z) of the wire, the confinement frequency, and the strength of the electric and magnetic fields has an important effect on the transverse magnetoconductivity and the relaxation rates, as well as the MPR effects for a Q1D quantum structure. The detailed discussion about these effects will be given explicitly in next two sections.

III. Electric-field-dependent magnetoconductivity

We want to evaluate the electric-field-dependent transverse magnetoconductivity $\sigma_{xx}(E)$ for the Q1DEG system subjected to the crossed electric $\vec{E}(\parallel\hat{x})$ and magnetic $\vec{B}(\parallel\hat{z})$ fields by taking into account the general expression for the nonlinear dc conductivity $\sigma_{i,j}(E)(i,j=x,y,z)$ given in Eq.(4.38) of Ref. 10 and considering the following matrix elements in the representation (2):

$$j_x=-(e/m^*)p_x, \tag{4}$$

$$|\langle\lambda|j_x|\lambda'\rangle|^2=(e\,l_B\,\tilde{\omega}_c/\sqrt{2})^2[(N+1)\delta_{\lambda',\lambda+1}+N\delta_{\lambda',\lambda-1}], \tag{5}$$

$$|\langle\lambda|\exp(\pm i\vec{q}\cdot\vec{r})|\lambda'\rangle|^2=|J_{N,N'}(x_\lambda,\pm q_x,x_{\lambda'})|^2|F_{nn'}(\pm q_z)|^2\delta_{k_y,k_y'\pm q_y} \tag{6}$$

$$|J_{NN'}(x_\lambda,\pm q_x,x_{\lambda'})|^2=|J_{NN'}(u)|^2=\frac{N_n!}{N_m!}e^{-u}u^{N_m-N_n}[L_{N_n}^{N_m-N_n}(u)]^2, \tag{7}$$

$$|F_{nn'}(\pm q_z)|^2=|F_{nn'}(t)|^2=\left[\frac{\sin(t)}{\cos(t)}\right]^2\frac{\pi^2nn'^2t^2}{t^2-[(\pi/2)(n-n'^2)]^2t^2-[(\pi/2)(n+n'^2)]^2}, \tag{8}$$

where j_x is the x component of a single-electron current operator and the Kronecker symbols $\delta_{\lambda',\lambda\pm 1}=\delta_{N',N\pm 1}\delta_{n',n}\delta_{k_y',k_y}$ denote the selection rules, which arise during the integration of the matrix elements with respect to each direction. $N_n=\min\{N,N'\}$, $N_m=\max\{N,N'\}$, $u=\tilde{l}_B^2 q_\perp^2/2$, $q_\perp^2=q_x^2+\tilde{b}^2 q_y^2$, $t=L_z q_z/2$, and $L_N^M(u)$ is an associated Laguerre polynomial. It should be noted that the general expression for the nonlinear dc conductivity σ_{ij} derived in Ref. 10 is the basis for the present theory and is strictly valid when the scatters(phonons) remain at equilibrium. The above equations except for Eq.(8) are similar to those of Vasilopoulos et al.[3] and Suzuki [8] obtained for the Q2D quantum-well structure. The main difference is that the effect of the confinement frequency is included in Eqs.(5), (6) and (7). The upper sin( ) in Eq.(8) is for n and n' both even or both odd, the lower cos( ) is for one of them even and the other odd; hence for interelectric subband scattering ($n'\neq n$), the term with cos( ) must be taken in Eq.(8). The overlap integrals $J_{NN'}$ and $F_{nn'}$ in Eq.(6) are, respectively, the quantities defined as

$$J_{NN'}(x_\lambda,\pm q_x,x_{\lambda'})=\int_{-\infty}^{\infty}\phi_N^*(x-x_\lambda)e^{\pm iq_x x}\phi_{N'}(x-x_{\lambda'})dx \qquad (9)$$

$$F_{nn'}(\pm q_z)=(2/L_z)\int^{L_{z0}} e^{\pm iq_z z}\sin(n\pi z/L_z)\sin(n'\pi z/L_z)dz \qquad (10)$$

For the calculation of the electric-field-dependent transverse magnetoconductivity $\sigma_{xx}(E)$ for the Q1D quantum structure, we apply the general expression for the electric-field-dependent dc conductivity $\sigma_{ij}(E)\,(i,j=x,y,z)$ given in Eq.(4.38) of Ref. 10 to the Q1D quantum structure modeled in Sec.II by using the selection rules of Eq.(5) and replacing the λ_1 and λ_2 states of Eq.(4.38) in Ref. 10 with the representation (2). Then, $\sigma_{xx}(E)$ can be easily obtained by

$$\sigma_{xx}(E)=\frac{e^2\tilde{l}_B^2\tilde{\omega}_c}{V}\sum_\lambda(N+1)[f(E^0_{\lambda+1})-f(E^0_\lambda)]A_{\lambda+1,\lambda}(E), \qquad (11)$$

where the summation over λ means $\sum_\lambda=\sum_{N,n,k_y}$ since $|\lambda\rangle\geq|N,n,k_y\rangle$, $f(E^0_\lambda)$ is a Fermi-Dirac distribution function with $E^0_\lambda=(N+1/2)\hbar\tilde{\omega}_c+\varepsilon_n(k_z)+\hbar^2k_y^2/2\tilde{m}$, and the spectral density $A_{\lambda+1,\lambda}(E)$ is given by

$$A_{\lambda+1,\lambda}(E)=\frac{\Gamma_{\lambda+1\lambda}(E)}{[E_{\lambda+1}-E_{\lambda}+\tilde{\nabla}_{\lambda+1\lambda}(E)]^2+[\Gamma_{\lambda+1\lambda}(E)]^2}\ . \qquad (12)$$

Here the quantities Γ and $\tilde{\nabla}$, which appear in terms of the collision broadening due to the electron-background (phonon and/or impurity) interaction, play role of the width and the shift in the spectral line shape, respectively. We assume Γ, $\tilde{\nabla} << \hbar\tilde{\omega}_c(=E_{\lambda+1}-E_{\lambda})$ and shift zero to observe the oscillatory behavior of MPR as some other authors did [3,6,8], the spectral densities in Eq.(12) can then be approximated as $\Gamma_{\lambda+1\lambda}/(\hbar\tilde{\omega}_c)^2$. To express the nonlinear dc magnetoconductivity of Eq.(11) in simpler forms, we assume that the f s in Eq.(11) are replaced by the Boltzmann distribution function for nondegenerate semiconductor, i.e., $f(E^0_{\lambda})=f_{N,n}(k_y)\approx\exp[\beta_e(\zeta-E^0_{N,n,k_y})]$, where $\beta_e=1/k_BT_e$ with k_B being Boltzmann's constant, T_e the hot-electron temperature and ζ the Fermi energy. Then, we can further perform the sum over N (if N is large) by writing $\sum N\exp(-\alpha N)=-\frac{\partial}{\partial\alpha}\sum\exp(-\alpha N)$ and summing the geometric series, and carrying out the one summation with respect to k_y in $\sum_{N,n,k_y}$ in terms of the following relation [8] :

$$\sum_{k_y}(\cdots)\rightarrow(L_y/2\pi)\int_{-L_x/2\tilde{b}\tilde{l}^2_B-eE/\hbar\omega_c}^{L_x/2\tilde{b}\tilde{l}^2_B-eE/\hbar\omega_c}dk_y(\cdots) \qquad (13)$$

since the upper and lower limits are obtained from the facts that electrons should be within the crystal dimensions in the x direction, i.e., $-L_x/2\le x\le L_x/2$ and that functions $\Phi_N(x-x_{\lambda})$ are centered at $x_{\lambda}=-\tilde{b}\tilde{l}_B^2(k_y+eE/\hbar\omega_c)$. Thus, we obtain

$$\sigma_{xx}(E)\approx\frac{e^2N_s^{1D}}{m^*\hbar V\tilde{\omega}_c^2}erf\left[\sqrt{\frac{\hbar^2}{2\tilde{m}k_BT_e}}\frac{L_x}{2\tilde{b}\tilde{l}_B^2}-\frac{eE}{\hbar\omega_c}\right]\Gamma_{\lambda+1\lambda}(E) \qquad (14)$$

where $erf[z]=(2/\sqrt{\pi})\int_0^z \exp[-t^2]dt$ indicates the error function. To derive Eq.(14) we utilized the electron density[6] given as $N_s^{1D}=\sqrt{\tilde{m}L^{2\prime}/8\pi\hbar^2\beta_e}\sum_n \exp[\beta_e(E_F-\varepsilon_n(z))]/\sinh(\beta_e\hbar\tilde{\omega}_c/2)$. According to Vasilopoulos et al. [6], the upper (lower) bound of Eq.(13) is given by the positive (negative) infinity. In that case, the error function should be replaced by 1. It should be noted that $\Gamma_{\lambda+1\lambda}$ is refered to as the relaxation rate associated with the states $\lambda+1$ and lambda since the field-dependent relaxation (or collision) time $\tau(E)$ can be estimated from $\tau(E)\approx\hbar/\Gamma(E)$, and also it depends on the confinement frequency (Ω), the wire width(L_z), and the strength of the applied electric and magnetic fields since these effects are included in the eigenstate energy E_λ. As seen from Eq.(14), the field-dependent transverse magnetoconductivity $\sigma_{xx}(E)$ is closely related to the relaxation rate $\Gamma_{\lambda+1\lambda}(E)$. Thus, the electronic transport properties (e.g., electronic relaxation processes, ICFE, ordinary and hot-electron magnetophonon resonances, etc.) in the Q1D quantum structures can be studied by examining the behavior of $\Gamma(E)$ as a function of relevant physical parameters introduced in the theory. The general form of the field-dependent relaxation rate Γ is given in Ref.10, which is obtained for both the weak-coupling and the strong-coupling cases with respect to the electron-background (phonon and/or impurity) interaction. In this paper, we will use the general form of the field-dependent relaxation rate Γ for the weak-coupling case since that for the strong-coupling case is so complicated that we cannot evaluate the relaxation rate analytically. However, any problem such as the delta-function singularities in Γ, appearing when the general relaxation rate Γ for the weak-coupling case is taken into account, will be removed by introducing an appropriate parameter. The detailed description will be given in the next section.

The general form of the field-dependent $\Gamma(E)$ for an electron-phonon system is given by Eq.(4.39) of Ref.10. Using the representation given by Eq.(2), the Q1D version of this quantity associated with the electronic transition between the state $|\lambda+1\rangle$ and $|\lambda\rangle$ can be evaluated as

$$\begin{aligned}\Gamma_{\lambda+1\lambda}(E) = \pi\sum_{q}\sum_{k_y}\sum_{(N',n')\neq(N,n)} &|C(q)|^2|J_{NN'}(u)|^2|F_{nn'}(q_z)|^2\\ &\{(N_{\vec{q}}+1)\delta[E_{N+1n}(k_y)-E_{N'n'}(k_y-q_y)-\hbar\omega_{\vec{q}}]\\ &+N_{\vec{q}}\delta[E_{N+1n}(k_y)-E_{N'n'}(k_y+q_y)+\hbar\omega_{\vec{q}}]\}\\ +\pi\sum_{q}\sum_{k_y}\sum_{(N',n')\neq(N+1,n)} &|C(q)|^2|J_{N'N+1}(u)|^2|F_{n'n}(q_z)|^2\\ &\{(N_{\vec{q}}+1)\delta[E_{N'n'}(k_y-q_y)-E_{Nn}(k_y)+\hbar\omega_{\vec{q}}]\\ &+N_{\vec{q}}\delta[E_{N'n'}(k_y+q_y)-E_{Nn}(k_y)-\hbar\omega_{\vec{q}}]\}\end{aligned}\quad, \qquad (15)$$

where N' and n' indicate the intermediate localized Landau and subband level indices, respectively and $N_{\vec{q}}$ is the equilibrium distribution function for a phonon with momentum $\hbar\vec{q}$ and energy $\hbar\omega_{\vec{q}}$: $N_{\vec{q}}=[\exp(\beta\hbar\omega_{\vec{q}})-1]^{-1}$. Here $\beta=1/k_BT$ with T being the (lattice) temperature. It should be noted that Eq.(15) is valid irrespective of both the strength of the electric fields and the magnitude of the confinement frequency included in the eigenstate energies E_λ of the Hamiltonian h_{eE}. The δ functions in Eq.(15) express the law of energy conservation in one-phonon collision (absorption and emission) processes, where the effect of the electric field (ICFE) is entered exactly through the exact eigenstate energy E_λ of an electron [cf. Eq.(3)]. The strict energy-conserving δ functions in Eq.(15) imply that when the electron undergoes a collision by absorbing the energy from the field. This in fact leads to electric-field-induced resonance effects due to the Landau levels or the subband levels. This EFIMPR effect for nonpolar phonon scattering will be shown explicitly in Sec.IV. If we neglect the electric field effect in Eq.(15), i.e., $E\to 0$, this reduces to the usual case [6,9] where collisions are instantaneous and the result exhibits the usual phonon emission and absorption processes, giving rise to the ordinary MPR of Q1D quantum structure for the Ohmic (weak-field) case seen in Refs.6 and 9, where the ICFE is not effective.

Ⅳ. Electric-field-induced magnetophonon resonances

For the evaluation of the field-dependent relaxation rate $\tilde{\Gamma}_{\lambda+1\lambda}$ for a specific electron-phonon interaction, we need the Fourier component of interaction potential

$C(q)$. To make the practical calculation easier, we will take the Fröhlich interaction potential given by [12] $|C(q)|^2 = A/Vq^2$ with $A = 4\pi\alpha\hbar(\hbar\omega_{LO})^{3/2}/(2m^*)^{1/2}$ and α being the dimensionless electron-phonon (polaron) coupling constant, where the assumption that the phonons are dispersionless (i.e., $\hbar\omega_{\vec{q}} \approx \hbar\omega_{LO} \approx$ constant, where ω_{LO} is the polar-LO-phonon frequency) and bulk (i.e., three-dimensional) was made. We also need the following matrix element [8] :

$$I_{nn'}(q'_\perp) = \int_{-\infty}^{\infty} \frac{|F_{nn'}(q_z)|^2}{q_z^2 + q'^2_\perp} dq_z$$

$$= \pi L_z \left[\frac{1+\delta_{nn'}}{[(n-n'^2\pi^2 + q'^2_\perp L_z^2]} + \frac{1}{[(n+n'^2\pi^2 + q'^2_\perp L_z^2]}\right](1 - S_{nn'}) \tag{16}$$

and

$$S_{nn'} = \frac{q'_\perp L_z[1 \mp \exp(-q'_\perp L_z)]}{[(n-n'^2\pi^2 + q'^2_\perp L_z^2][(n+n'^2\pi^2 + q'^2_\perp L_z^2]} \times \frac{32\pi^2 n^2 n'^2}{\{[(n-n'^2\pi^2 + q'^2_\perp L_z^2] + (1+\delta_{nn'})[(n+n'^2\pi^2 + q'^2_\perp L_z^2]} \tag{17}$$

where $q'_\perp = \sqrt{q_x^2 + q_y^2}$. The upper sign in Eq.(17) should be taken if n and n' are both even or odd, and the lower sign if n is odd and n' is even or vice versa. For the case that the electric field is applied in the x direction, Eq.(16) can be approximated [3,8] as

$$I_{nn'}(q'_\perp) \approx \frac{\pi}{q'^2_\perp} L_z(2 + \delta_{nn'}) \tag{18}$$

for $q'_\perp \gg q_z$ since we can expect the largest contribution to the current comes from the processes involving large momentum transfer in the x direction, i.e., those processes with larger q_x and consequently large $q'_\perp$ and small q_z. Therefore, Eq.(18), rather than Eq.(16), will be utilized in the practical calculation.

Let us now calculate the field-dependent relaxation rate $\Gamma_{\lambda+\mu}$ of Q1DEG for the polar-LO-phonon scattering. Transforming the sum over $\vec{q}$ in Eq.(15) into the integral form in a usual way and considering Eqs.(3), and (18), the Q1D version of the relaxation rates $\Gamma_{\lambda+1\lambda}$ for LO-phonon scattering can be written as

$$\Gamma_{\lambda+1\lambda} = \frac{\pi A}{(2\pi)^3}\sum_{k_y}\sum_{NN'}\int_{-\infty}^{\infty}dq_x\int_{-\infty}^{\infty}dq_y I_{nn'}(q'_\perp)|J_{NN'}(u)|^2$$
$$\times\{(N_0+1)\delta[(N-N'+1)\hbar\tilde{\omega}_c+\varepsilon^-+\hbar\omega_{nn'}-\hbar\mathfrak{b}^2V_dq_y-\hbar\omega_{LO}]$$
$$+N_0\delta[(N-N'+1)\hbar\tilde{\omega}_c-\varepsilon^++\hbar\omega_{nn'}+\hbar\mathfrak{b}^2V_dq_y+\hbar\omega_{LO}]\}$$
$$+\frac{\pi A}{(2\pi)^3}\sum_{k_y}\sum_{NN'}\int_{-\infty}^{\infty}dq_x\int_{-\infty}^{\infty}dq_y I_{nn'}(q'_\perp)|J_{NN'}(u)|^2$$
$$\times\{(N_0+1)\delta[(N'-N+1)\hbar\tilde{\omega}_c-\varepsilon^--\hbar\omega_{nn'}+\hbar\mathfrak{b}^2V_dq_y+\hbar\omega_{LO}]$$
$$+N_0\delta[(N'-N+1)\hbar\tilde{\omega}_c+\varepsilon^+-\hbar\omega_{nn'}-\hbar\mathfrak{b}^2V_dq_y-\hbar\omega_{LO}]\} \qquad (19)$$

where $\omega_{nn'}=(n^2-n'^2)\varepsilon_0/\hbar$, N_0 is the polar-LO-phonon distribution function given by $N_{\vec{q}}$ with $\omega_{\vec{q}}=\omega_{LO}$, and $\epsilon^{\pm}=(\hbar/2\tilde{m}\tilde{\omega}_c)(q_x^2\pm 2k_xq_x)$ It should be noted that the above equation is valid for the narrow and wide confinement frequency. The evaluation of Γ in Eq.(19) involves further integrations with respect to k_y, q_x and q_y in the Cartesian coordinates. The integral is very difficult to evaluate analytically. To simplify the calculations, we shall restrict ourselves to the case of the narrow confining frequencies such that $\Omega << \tilde{\omega}_c$. Then, an approximation can be made as follows: First, if the confining frequencies become narrower, the renormalized mass $\tilde{m}$ will be larger. As a result, we can expect that the $\varepsilon^{\pm}$ terms is smaller than other terms within the delta function. Hence, in the following we will consider the case that the $\varepsilon^{\pm}$ terms in Eq.(19) can be neglected as Vasilopoulos et al.[6] did. Secondly, the integrand over q_x and q_y is combersome due to the factor $I(q'_\perp)$ and the matrix element $|J_{NN'}(u)|^2$. Therefore, another approximation is to replace $q'_\perp(=\sqrt{q_x^2+q_y^2})$ appearing in the factor $I(q'_\perp)$ by $q_\perp(=\sqrt{q_x^2+\mathfrak{b}^2q_y^2})$ since $\mathfrak{b}^2=1-\varepsilon'$ with $\epsilon'2/\tilde{\omega}_c^2 << 1$ is nearly equal to 1 for the narrow confining frequencies. Furthermore, we replace $\hbar\mathfrak{b}^2V_dq_y$ in the argument of the δ functions by the effective potential-energy difference $eE\Delta\bar{x}$ across the spatial extent $\Delta\bar{x}$ of a Landau state, where $\Delta\bar{x}$ is a constant of the order of the magnetic length l_B. With these approximations and Eq.(13) we can do the integral over k_y, q_x and q_y. Especially, the integral over the $\vec{q}$ space can be reduced to the integrals with respect to theta and $q_\perp$ (or u) in the cylindrical coordinates, where the θ integration gives 2π. To get the field-dependent relaxation rate given in a simple form, we assume[8] that N' is very large, we can, then, approximate $N'\pm1\approx N'$. Setting $N'-N=-P$ in the emission term and $N'-N=P$ in the absorption term, and noting [3,8] that $\int_0^\infty |J_{NN'}(u)|^2 u^{-1}du = 1/P$, $P=1,2,3,\cdots$, we obtain for the narrow confining frequencies as

$$\begin{aligned}\tilde{\Gamma}_{\lambda+1,\lambda}(E) \approx\ & 3\Lambda\sum_P (2N_0+1)\delta[P-\omega^*_{LO}/\tilde{\omega}_c]/P \\ & +2\Lambda\sum_{n'(\neq n)}\sum_P\{(N_0+1)\delta[P-(\omega^*_{LO}-\omega_{NN'})/\tilde{\omega}_c] \\ & +N_0\delta[P-(\omega^*_{LO}+\omega_{NN'})/\tilde{\omega}_c)]\}/P \end{aligned} \tag{20}$$

where $\Lambda = AL_xL_y\omega_c^2/(4\pi L_z m^* \tilde{\omega}^3)$ and $\omega^*_{LO}=\omega_{LO}+eE\Delta\bar{x}/\hbar$. Furthermore, we see that the above equation gives rise to the oscillatory behavior by applying Poisson's summation formula ^{13} for the sum $\sum_P$:

$$\begin{aligned}\Gamma_{\lambda+1,\lambda}(E) \approx\ & 3\Lambda[\frac{(2N_0+1)}{x}[1+2\sum_{s=1}^{\infty}\cos(2\pi s x)]] \\ & +2\Lambda\sum_{n'(\neq n)}[\frac{(N_0+1)}{x(1-y)}[1+2\sum_{s=1}^{\infty}\cos(2\pi s x(1-y))] \\ & +\frac{N_0}{x(1+y)}[1+2\sum_{s=1}^{\infty}\cos(2\pi s x(1+y))]] \end{aligned} \tag{21}$$

where $x=\omega^*_{LO}/\tilde{\omega}_c$ and $y=\omega_{NN'}/\omega^*_{LO}$. Note that the electric-field-dependent transverse magnetoconductivity Eq.(14) associated with the relaxation rate shows the resonant behaviors: electric-field-induced magnetophonon resonances at $P\tilde{\omega}_c=\omega^*_{LO}$

and at $P\tilde{\omega}_c = \omega_{LO}^* \pm \omega_{NN'}$ (P is an integer). The above conditions for the EFIMPR give the resonance magnetic fields (i.e., the EFIMPR peak positions at) at B_p, B_p^+, and B_p^- :

$$B_p = \sqrt{[B_0 + (m^* \Delta\bar{x}/\hbar)E]^2/p^2 - (m^*\Omega/e)^2} \qquad (22a)$$

$$B_p^{\pm} = \sqrt{[B_0 + (m^* \Delta\bar{x}/\hbar)E \pm (m^*\omega_{NN'}/e)]^2/P^2 - (m^*\Omega/e)^2} \quad (n' \neq n) \qquad (22b)$$

where $B_0(=m^*\omega_L/e)$ is the fundamental field for the MPR. The origin of the appearance of the subsidiary peaks in the Q1D quantum structure is mainly due to the interelectric subband scattering (i.e, interelectric nonresonant virtual subband transitions) as in the case of Q2D quantum structure[8]. We can also see that the contribution to the MPR peak positions of the wire width(L_z) depends on the type of the interelectric virtual subband transitions. Eqs.(22a) and (22b) are in agreement with the result of Suzuki[8] obtained in the Q2D quantum structure if the confining frequencies Ω approach zero. Furthermore, if the strength of electric field E and $\omega_{NN'}$ terms are neglected, Eqs.(22a) and (22b) are identical with the result of Vasilopoulos et al.[6] predicted in the linear regime of the Q1D quantum structure. It should be noted that the relaxation rate for polar-LO-phonon scattering diverges whenever the above conditions are satisfied. These divergences may be removed by including higher-order electron-phonon scattering terms or by inclusion of the fluctuation effects of the center-of-mass[14]. The simplest way to avoid the divergences is the each delta function in Eq.(20) is approximated by Lorentzians of width and shift zero by introducing a width parameter γ. Employing this collision-broadening model [3,5,8], applying Poisson's summation formula [13] for the $\sum_p$ in Eq.(20), and taking into account the following property[8,15] :

$$\Psi(a, b) = 1 + 2\sum_{s=1}^{\infty} e^{-2\pi sa}\cos(2\pi sb) = \frac{\sinh(2\pi a)}{\cosh(2\pi a) - \cos(2\pi b)}, \quad (a>0) \qquad (23)$$

we then obtain

$$\tilde{\Gamma}_{\lambda+1,\lambda}(E) \approx 3\Lambda(2N_0+1)\frac{x}{x^2+(\gamma_1/\hbar\tilde{\omega}_c)^2}\Psi(\frac{\gamma_1}{\hbar\tilde{\omega}_c},x)$$

$$+2\Lambda(N_0+1)\sum_{n'\neq n}[\frac{x(1-y)}{x^2(1-y)^2+(\gamma_2/\hbar\tilde{\omega}_c)^2}\Psi(\frac{\gamma_2}{\hbar\tilde{\omega}_c},x(1-y))$$

$$+2\Lambda N_0\sum_{n'\neq n}[\frac{x(1+y)}{x^2(1+y)^2+(\gamma_3/\hbar\tilde{\omega}_c)^2}\Psi(\frac{\gamma_3}{\hbar\tilde{\omega}_c},x(1+y)) \quad (24)$$

Equation (24) gives a general description of high-field magnetophonon oscillations in the Q1D quantum structure with the narrow confining frequencies. We can see that if the confining frequencies have been neglected, Eq.(24) reduces to the result of Suzuki[8] obtained for the Q2D quantum structure. Therefore, as in the case of the Q2D quantum structure[8], we can obtain the field-dependent relaxation rate for the narrow and wide wire width as

$$\Gamma_{0\lambda+1\lambda}(E)\approx 3\Lambda(2N_0+1)\frac{\omega^*_{LO}/\tilde{\omega}_c}{(\omega^*_{LO}/\tilde{\omega}_c)^2+(\gamma/\hbar\tilde{\omega}_c)^2}\Psi[\frac{\gamma}{\hbar\tilde{\omega}_c},\frac{\omega^*_{LO}}{\tilde{\omega}_c}] \quad (25)$$

$$\Gamma_{\lambda+1\lambda}(E) \approx 3\Lambda(2N_0+1)\frac{\omega^*_{LO}/\tilde{\omega}_c}{(\omega^*_{LO}/\tilde{\omega}_c)^2+(\gamma/\hbar\tilde{\omega}_c)^2}\Psi[\frac{\gamma}{\hbar\tilde{\omega}_c},\frac{\omega^*_{LO}}{\tilde{\omega}_c}]$$

$$+2\Lambda(N_0+1)\sum_{\pm}\frac{\omega^{\pm}_1/\tilde{\omega}_c}{(\omega^{\pm}_1/\tilde{\omega}_{c^2})+(\gamma/\hbar\tilde{\omega}_c)^2}\Psi[\frac{\gamma}{\hbar\tilde{\omega}_c},\frac{\omega^{\pm}_1}{\tilde{\omega}_c}]$$

$$+2\Lambda N_0\sum_{\pm}\frac{\omega^{\pm}_2/\tilde{\omega}_c}{(\omega^{\pm}_2/\tilde{\omega}_c)^2+(\gamma/\hbar\tilde{\omega}_c)^2}\Psi[\frac{\gamma}{\hbar\tilde{\omega}_c},\frac{\omega^{\pm}_2}{\tilde{\omega}_c}] \quad (26)$$

where $\omega^{\pm}_1=\omega^*_{LO}+(1\pm 2n)\varepsilon_0/\hbar$ and $\omega^{\pm}_2=\omega^*_{LO}-(1\pm 2n)\varepsilon_0/\hbar$. it should be noted that Eq.(25) is valid for the narrow confining frequencies and the narrow wire width. In this case, Eq.(25) shows that those subsidiary EFIMPR peaks (due to the interelectric subband transitions) do not appear, and the period of the oscillation is given under the condition of $\omega^*_{LO}/\tilde{\omega}_c=P$ and is determined by the strength of magnetic and electric fields, as well as the confining frequencies. We see that the subsidiary (EFIMPR) peaks appear at $P\tilde{\omega}_c=\omega^{\pm}_i$ $(i=1 \text{ and } 2)$ and that the position of these subsidiary peaks and the period of additional oscillations in Eq.(26) are sensitive to the wire width. For $P\tilde{\omega}_c=\omega_{LO}$, the factors cos() in Eq.(26) become $\cos(2\pi seE\Delta\bar{x}/\hbar\tilde{\omega}_c)$ for the first term, which is identical result with Eq.(25), and $\cos 2\pi s[eE\Delta\bar{x}+(1\pm2n)\epsilon_0]/\hbar\tilde{\omega}_c$ and $\cos 2\pi s[eE\Delta\bar{x}-(1\pm2n)\epsilon_0]/\hbar\tilde{\omega}_c$ for the second and the third terms, respectively. Hence, by varying the electric field at the same magnetic field, the ordinary MPR maxima (at $\omega_{LO}=P\tilde{\omega}_c$) in the conductivity can evolve to minima and vice versa. We can also see that the exponential factors play a role of the effect of field-induced-collision damping due to the combined effect of scatterings (or collisions) and electric fields (ICFE) since the γ's in Eqs.(25) and (26) are generally dependent on the field strength E, and that these parameters such as strong electric and magnetic fields, and the wire width as well as the confinement frequency give an direct influence on the effect of field-induced-collision damping. For vanishingly small electric field (i.e., $E\rightarrow 0$), Eq.(25) leads to the ordinary MPR of the Q1D quantum structure at $\omega_{LO}=P\tilde{\omega}_c$, and is similar to the result of Vasilopoulos et al.[6] obtained for the linear (weak field) case of the Q1D quantum structure. The main diference depends on whether the Lorentzian form factor is included or not, which is mainly due to the approximation of $I_{NN'}(q'_\perp)$ in Eq.(19).

V. Concluding remarks

In this paper, starting from the field-dependent conductivity formula defined in the Ohm's law form of the nonlinear current density, we have presented a theory of hot-electron MPR and investigated the physical characteristics of the hot-electron MPR effects in the Q1D quantum structure. The origin of this formalism [10] dates back to the discovery of the theory of nonlinear static conductivity. On the basis of this formalism, The field-dependent relaxation rate for the weak-coupling case has been utilized with respect to the electron-LO-phonon interaction and its behavior (transport process) has been discussed in connection with the EFIMPR effect. It is shown from Eqs.(14) and (24) that the relaxation rates $\Gamma_{\lambda+\mu}(E)$ is inversely proportional to the wire width(L_z) and hence that $\sigma_{xx}(E)$ has a $1/L_z^2$ dependence. These results are identical with the theoretical results obtained for the Q2D quantum structure. The field-dependent transverse magnetoconductivity and the relaxation rate show the Q1D

EFIMPR at $P\tilde{\omega}_c=\omega^*_{LO}$ and $\omega^*_{LO}\pm\omega_{NN'}$. Here P is an integer. Therefore, the EFIMPR peak positions are closely related to the confinement frequency (Ω) and the strength of the electric field(E), as well as the wire width(L_z). The effect of electric fields in the scattering processes of the Q1D quantum structure is to shift the MPR peak positions to higher magnetic fields, while the effect of confinement due to the confinement frequency in the x direction is to change the MPR peak positions to lower magnetic field. It is noted that our result for the relaxation rate and the field-dependent transverse magnetoconductivity is valid for the narrow confinement frequency and is far from being rigorous for the wide confinement frequency since our result is tied to the approximations: the $\varepsilon^{\pm}$ terms of Eq.(19) have been neglected for the narrow confinement frequency and another approximation has been made by replacing $q'_{\perp}(=\sqrt{q_x^2+q_y^2})$ appearing in the factor $I(q'_{\perp})$ by $q'_{\perp}(=\sqrt{q_x^2+b^2q_y^2})$ in order to get tractable expressions within the integration over $\vec{q}$ of Eq.(19). Furthermore, we have not taken into account any modification of the electron-phonon interaction brought about by the confinement of phonons[we used the interaction for bulk phonons] and any influence that surface roughness might have on the effect as some author did. However, we can expect that our result makes it possible to understand qualitatively the physical characteristics on the EFIMPR effect of the Q1D quantum structure. Unfortunately, we are not aware of any relevant experimental work to compare our theory with. Therefore, to test the validity of this prediction, new experiments are needed.

References

[1] R.J. Nicholas, Prog. Quantum Electron. 104, 1 (1985).
[2] L. Eaves, P.S.S. Guimaraes, J.C. Portal, T.P. Pearsall, and G. Hill, Phys. Rev. Lett. 53 608 (1984); J. Phys. C 17 , 6177 (1984).
[3] P. Vasilopoulos, M. Charbonneau, C.M. Van Vliet, Phys. Rev. B 35 , 1334 (1987).
[4] P. Warmenbol, F.M. Peeters, and J.T. Devreese, Phys. Rev. B 39 7821 (1989); 37, 4694(1988).
[5] N. Mori, H. Murata, K. Taniguchi, and C. Hamaguchi, Phys. Rev. B 38 , 7622 (1988).
[6] P. Vasilopoulos, P.Warmenbol, F.M. Peeters, and J.T. Devreese, Phys. Rev. B 40 , 1810(1989).
[7] N. Mori, H. Momose, and C. Hamaguchi, Phys. Rev. B 45 , 4536(1992).
[8] A. Suzuki, Phys. Rev. B 45 , 6731(1992).
[9] J.Y. Ryu and R.F. O'Connell, Phys. Rev. B (Accepted for publication).
[10] J.Y. Ryu and S.D. Choi, Phys. Rev. B 44 , 11328(1992).
[11] J.R. Barker, J. Phys. C5, 1657 (1972).
[12] R. F. Wallis and M. Balkanski, Many-Body Aspect of Solid State Spectroscopy North-Holland, Amsterdam, 1986), Chap. 7.
[13] J.M. Ziman, Principles of the Theory of Solids(Cambridge University Press, Cambridge, 1972), P.319.
[14] G.Y. Hu and R.F. O'Connell, Phys. Rev. B 40 , 11701 (1989).
[15] I.S. Gradshteyn and I.M. Ryzhik, Table of Integrals, Series, and Products(Academic, New York, 1965).