INTRODUCTION TO AUCTIONS

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목 차

Ⅰ. Introduction

In this paper, we introduce a basic theory of auctions as a novice guide to the literature. The practices of Auctions have an ancient root. For example, some women were auctioned off in Babylonia to be wives around the fifth century B.C. In china, as early as 7th century A.D. the belongings of deceased Buddhist monk were sold out at auctions. Currently Government uses some auction forms to sell right for natural resources and some treasury bills. Some private agents acquire government contracts by procurement auctions. Firms employ the procurement auctions to buy inputs or subcontract work. In these cases, the auctioneer is seeking a lower price than a high price. Every kind of commodities such as houses, cars, agricultural products and livestock art and antiques are being sold out by auctions. A lot of the world's most important markets are auction markets such as oil fields and 3G mobile

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phone markets. The range of items sold by auction has been greatly increased by e-commerce.

As auction markets become an increasingly important part of the economy, the theory of auction market design has become the basis of much fundamental theoretical work not directly related to auctions. So, auction theory has become the most successful branches of economics, ever since the seminal paper of Vickrey. (1960). The theory has developed rapidly, and is increasingly being looked to for assistance in practical applications.

The paper is organized as follows. In Section II, we introduce the basic terminology of auction theory including several auction formats of common and private value auctions and solving procedures of independent private value auctions. In addition to that, we explained the importance of auctions in theoretical, empirical and practical reasons. In Section III, we have shown the celebrated result of Revenue Equivalence Theorem. Finally, we make a brief summary for some extensions of ongoing research of this area in section IV.

II. Auction Theory

1. The practical and theoretical importance of auctions

Auctions are one of the oldest form of markets, dating back to at least 500 B.C. Today, all sorts of commodities are sold using auctions.

Economists became interested in auctions in the early 1970s when the OPEC oil cartel raised the price of oil. The U.S Department of the Interior decided to hold auctions to sell the right to drill in coastal areas that were expected to contain vast amounts of oil. The government asked economists how to design these auctions and private firms hired economists as consultants to help them design a bidding strategy. This effort prompted considerable research in auction design and strategy.

More recently, The Federal Communications Commission (FCC) decided to auction off parts of the radio spectrum for use by cellular phones, personal digital assistants, and other communication devices. Economists played a major role in the design of both the auctions and the strategies used by the bidders. These auctions were hailed as very successful public policy, resulting in revenues to the U.S government of over twenty-three billion dollars to date.

Other countries have also used auctions for privatization projects. For example, Australia sold off several government-owned electricity plants, and New Zealand auction off parts of its state-owned telephone system.

Consumer-oriented auctions have also experienced something of a renaissance on the Internet. There are hundreds of auctions on the Internet, selling collectibles, computer equipment, travel services, and other items. OnSale claims to be largest, reporting over forty-one million dollars worth of merchandise sold in 1997. And the growth of e-commerce has led to many business-to-business auctions for goods whose trade was previously bilaterally.[1)]

Auction theory is important for practical, empirical, and the theoretical reasons.

2. Classification of Auctions

Auctions are classified by two considerations first, what is the nature of the good that is being auctioned, second, what are the rules?

2.1. What is the nature of the good?

1) PRIVATE - VALUE AUCTIONS

In this case, each bidder knows how much she values the object for sale, but her value is private information to herself. Prior to the innovative work

1) Hal R.Varian, 「Intermediate Microeconomics」, pp.306-307

of Milgrom and Weber (1982), researchers in auction market design assume that the estimates of private values of the object are determined independently. According to Milgrom and Weber, this independent private value model works in case the object being sold is non-durable good; in case of durable good such as painting, the independent private value (IPV) assumption may not be valid. They argued that IPV model rules out the possibilities of resale of the object, some prestige value in owning that object, and some doubt about the authenticity of the object. In this line of argument they proposed that valuations of the object among bidders might be affiliated especially for the durable good.

2) COMMON - VALUE AUCTIONS

The actual value is the same for everyone, but bidders have different private information about what that value actually is. For example, the value of an oil-lease depends on how much oil is under the ground, and bidders may have access to different geological signals about that amount. In this case a bidder would change her estimate of the value if she learnt another bidder's signal, in contrast to the private-value case in which her value would be unaffected by learning any other bidder's preferences or information.

2.2. What are the rules of bidding

1) ASCENDING - BID AUCTION (ENGLISH AUCTION)

This is the most prevalent form of bidding structure auction. In this auction, the auctioneer starts with a reserve price, which is the lowest price at which the seller of the good will part with it. The price is successively raised until only one bidder remains, and that bidder wins the object at the final price.

In this auction the price rises continuously while bidders gradually quit the auction. Bidders observe when their competitors quit, and once someone quits, she is not let back in. There is no possibility for one bidder to pre-empty the process by making a large jump bid. In other words bidders successively offer higher prices: generally each bid must exceed the previous bid by some minimal bid increment. When no participant is willing to increase the bid further, the item is awarded to the highest bidder.

2) DESCENDING - BID AUCTION (DUTCH AUCTION)

This auction's name is due to its use in the Netherlands for selling cheese and fresh flowers. This auction works in exactly the opposite way: the auctioneer starts at a very high price, and then lowers the price continuously. The first bidder who calls out that she will accept the current price wins the object at that price. In practice, the auctioneer is often a mechanical device like a dial with a pointer which rotates to lower and lower values as the auction progresses. Dutch auction can proceed very rapidly, which is one of their chief virtues.

3) FIRST - PRICE SEALED - BID AUCTION[2)]

In this auction each bidder independently submits a single bid, without seeing others' bids, and the object is sold to the bidder who makes the highest bid. The winner pays her bid (that is, the price is the highest or first price bid).

4) SECOND - PRICE SEALED - BID AUCTION[3)]

In this auction each bidder independently submits a single bid, without

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2) In Sealed-bid auction, each bidder writes down a bid on a slip of paper and seals it in an envelope. The envelopes are collected and opened, and the good is awarded to the person with the highest bid who then pays the auctioneer the amount that she bid. If there is a reserve price, and all bids are lower than the reserve price, then no one may receive the item. These auctions are commonly used for construction work. The person who wants the construction work done requests bids from several contractors with the understanding that the job will be awarded to the contractor with the lowest bid.

3) This auction is sometimes called a Vickrey auction after William Vickrey, who wrote the seminal paper (1961) on auction.

seeing others' bids, and the object is sold to the bidder who makes the highest bid. However, the price she pays is the second-highest bidder's bid, or second price.

Sometimes, the ascending and descending are referred to as open second-price and open first-price auctions.[4]

3. Solving Common Private Value Auctions[5]

In all our auctions there are n participants and each participant has a valuation v_i and submits a bid b_i.

For example, two bidders are trying to purchase the same item at a sealed bid auction. The players simultaneously choose b_1 and b_2 and the good is sold to the highest bidder at his bid price (in the case of a tie a fair coin is flipped to determine the winner. Assume coin flip if $b_1 = b_2$). Suppose that the players' utilities are

$$u_i(b_i, b_{-i}) = \begin{cases} v_i - b_i & \text{if } b_i > b_{-i} \\ \frac{1}{2}(v_i - b_i) & \text{if } b_i = b_{-i} \\ 0 & \text{if } b_i < b_{-i} \end{cases}$$

Assume each has a prior that his rival's valuation is uniform on [0,1] and that this is common knowledge.

Let's expand definition a little bit.

4) In the descending auction each bidder must choose a price at which she will call out, conditional on no other bidder having yet called out; and the bidder who chooses the highest price wins the object at the price she calls out. Thus this game is strategically equivalent to the first price sealed-bid auction. That is, the set of strategies available to a player is the same in the descending auction as in the first-price sealed bid auction. Choosing any given bid yields the same payoffs in the games as a function of the other players' bids.

5) Markus M. Mobius, Game theory lecture 16, pp.3-5

The rules of the auction determine the probability $q_i(b_1, \ldots, b_n)$ that agent i wins the auction and the expected price $p_i(b_1, \ldots, b_n)$ which he pays.

Her utility is simple[6] $u_i = q_i v_i - p_i$

3.1. First-Price Auction

We focus on monotonic equilibria $b_i = f_i(v_i)$, f_i is strictly increasing. For simplicity, we work just with two bidders and values are drawn i.i.d. from the uniform distribution over [0,1]

The probability of player i winning the auction with bidding b is

$$prob(f_j(v_j) \leq b) = prob(v_j \leq f_j^{-1}(b)) = F(f_j^{-1}(b)) = f_j^{-1}(b)$$

The last equation follows because F is the uniform distribution.

The expected utility from bidding b is therefore:

$$prob(f_j(v_j) \leq b)(v_i - b) = f_j^{-1}(b)(v_i - b)$$

The agent will choose b to maximize this utility. Differentiate with respect to b and use the first-order condition:

$$\frac{1}{f_j'(f_j^{-1}(b))}(v_i - b) - f_j^{-1}(b) = 0$$

Focus on symmetric equilibria such that $f_i = f_j$. In equilibrium $b = f_i(v_i)$ such that:

$$\frac{1}{f'(v_i)}(v_i - f(v_i)) - v_i = 0$$

This is a differential equation and can be rewritten as follows:

6) The agent is risk-neutral.

$$v_i = v_i\ f'(v_i) + f(v_i)$$

Integrate both sides and get:

$$\frac{1}{2} v_i^2 + K = v_i f(v_i)$$

Where is a constant. Finally:

$$f(v_i) = \frac{1}{2} v_i + \frac{K}{v_i}$$

Check $f(0)=0$. Because the player with a zero valuation should never bid positive amounts. Hence $K=0$ is the only possible solution.

If you solve this exercise more generally for n bidders you get the following bidding function(uniform distribution):

$$f(v_i) = \frac{n-1}{n} v_i$$

As you increase the number of bidders there is more competition for the good and players have to make higher bids. Note that all bidders shade down their bid otherwise they would not make profits.

3.2. All pay Auction[7)]

We assume the same condition in the first condition
The corresponding utility function is:

$$prob(f_j(v_j) \le b) v_i - b = f_j^{-1}(b) v_i - b$$

The corresponding differential equation is :

7) In the all pay auction all bidders pay and the highest bidder wins.

$$\frac{1}{f(v_i)} v_i - 1 = 0$$

Thus the only solution is:

$$f(v_i) = \frac{1}{2} v_i^2$$

The general solution for n bidders is :

$$f(v_i) = \frac{n-1}{n} v_i^n$$

3.3. Second - Price Auction

This auction is different because it has a much more robust solution. In the second-price auction with private values bidding one's own valuation is a weakly dominant strategy. This means that no matter what the other players do you can never do worse by bidding your own valuation $b_i = v_i$.[8]

In other words, in this case it is clearly a weakly dominant strategy to stay in the bidding until the price reaches your value, that is, until you are just indifferent between winning and not winning. The next-to-last person will drop out when her value is reached, so the person with the highest value will win at a price equal to the value of the second-highest bidder. Furthermore a little reflection shows that in a second-price auction it is optimal for a player to bid her true value, whatever other players do. "truth telling" is a weakly dominant strategy equilibrium(and also a Nash equilibrium), so here, the person

8) Proof : denote the highest bid of all the other players except i by $\hat{b}$. Can i gain by deviating from bidding $b_i = v_i$? Assume that i bids higher such that $b_i > v_i$. This will only make a difference to the outcome of the auction for i if $v_i < \hat{b} < b_i$ in which case i will win the object now with the higher bid. But the utility from doing so is $v_i - \hat{b} < 0$ because i has to pay $\hat{b}$. Hence this is not profitable. Similarly, bidding below v_i is also non-profitable.

with the highest value will win at a price equal to the value of the second-highest bidder.

The intuition is that in the second-price auction my bid does not determine the price I pay. In the other two auction my bid equals my price. This makes me want to shade down my price. but if my bid does not affect the price but only my probability of winning then there is no reason to shade it down.

Ⅲ. Application to Economics

- Using auction theoretic tools in economics:
  The Revenue Equivalence Theorem[9]

How revenue does the auctioneer make from the auction?

The expected revenue is equal to n times the expected payment from each player. Hence to compare the revenue of different auction formats calculate the expected payment from each bidder with valuation v_i, two bidders and uniform distribution .

In the first price auction the expected payment from a player with valuation v_i is her bid $\frac{1}{2}v_i$ time the probability that she will win the auction which is v_i. So her expected payment is $\frac{1}{2}v_i^2$.

In the second price auction i pays the second highest bid if she wins. Since the other player bids her valuation is uniformly distributed over $[0, v_i]$. So the expected payment from i conditional of winning is $\frac{1}{2}v_i$. The expected payment is this conditional payment times the probability of winning, that is $\frac{1}{2}v_i^2$.

9) Markus M.Mobius, Game theory lecture 16, pp.5-7

In the all-pay auction player i always pays $\frac{1}{2}v_i^2$ which is equal to her expected payment.

Surprisingly, the revenue from all three auction formats is identical.
This is a special case of the revenue equivalence theorem.

1. The Revenue Equivalence Theorem[10)]

In the symmetric independent private value case all auctions which allocate the good to the player with the highest value for it and which give zero utility to a player with valuation $\underline{v}$ are revenue equivalent.

2. Proof of RET

Any auction mechanism which allocates the good will give player i with valuation v_i some surplus $S(v_i)$:

$$S(v_i) = q_i(v_i)v_i - p_i(v_i)$$

Where $q_i(v_i)$ is the probability of winning with valuation v_i and $p_i(v_i)$ is the expected price. We know by assumption that $S(\underline{v}) = 0$

Note that player i could pretend to be a different type $\tilde{v}$ and imitate $\tilde{v}$s strategy.

10) Paul Klemperer shows RET in his lecture. Assume each of a given number of risk-neutral potential buyers has a privately-known valuation independently drawn from a strictly-increasing atomless distribution, and that no buyer wants more than one of the k indivisible prizes. Then any mechanism in which (1)the prizes always go to the k buyers with the highest valuations and (2)any bidder with the lowest feasible valuation expects zero surplus, yields the same expected revenue and results in each bidder making the same expected payment as a junction of her valuation.

This deviation would give surplus:

$$\hat{S}(\tilde{v}, v_i) = q_i(\tilde{v})v_i - p_i(\tilde{v})$$

It has to be the case that i would not want to imitate type.
Hence we get :

$$\left.\frac{\partial S(\tilde{v}, v_i)}{\partial \tilde{v}}\right|_{\tilde{v}=v_i} = 0$$

we can calculate the derivative of the surplus function:

$$\frac{dS(v_i)}{dv_i} = \frac{d\hat{S}(v_i, v_i)}{dv_i} = \left.\frac{\partial S(\tilde{v}, v_i)}{\partial \tilde{v}}\right|_{\tilde{v}=v_i} + \left.\frac{\partial S(\tilde{v}, v_i)}{\partial v_i}\right|_{\tilde{v}=v_i} = q_i(v_i) = F(v_i)^{n-1}$$

Finally

$$S(v_i) = S(\underline{v}) + \int_{\underline{v}}^{v_i} F(t)^{n-1}dt = \int_{\underline{v}}^{v_i} F(t)^{n-1}dt$$

Hence the expected surplus for each player is identical across all auctions. But that also implies that the expected payments from each player are identical across auctions.

2. The Winner's Curse[11]

The winner's curse arises in first-price auctions with a common value environment, where the good that is being awarded has the same value to all bidders. However, each of the bidders may have different estimates of that value. To emphasize this, let us write the value of bidder i as $v+\varepsilon_i$ where

11) Hal R.Varian, Intermediate Microeconomics, pp.312

v is the true, common value and ε_i is the error term associated with bidder i's estimate.

Let's examine a sealed-bid auction in this framework. What bid should bidder i place? To develop some intuition, let's see what happens if each bidder bids their estimated value. In this case, the person with the highest value of ε_i, ε_{max}, gets the good, but as long as $\varepsilon_{max} > 0$, this person is paying more than v, the true value of the good. This is the so-called Winner's Curse. If you win the auction, it is because you have overestimated the value of the good being sold. In other words, you have won only because you were too optimistic!

The optimal strategy in a common -value auction like this is to bid less than your estimated value- and the more bidders there are, the lower you want your own bid to be. Think about it: if you are the highest bidder out your out of five bidders you may be overly optimistic, but if you are the highest bidder out of twenty bidders you must be super optimistic. The more bidders there are, the more humble you should be about your own estimates of the true value of the good in question.

The Winner's Curse seemed to be operating in the FCC's May 1996 spectrum auction for personal communications services. The largest bidder in that auction, Next Wave Personal Communications Inc. bid $4.2 billion for sixty-three licenses, winning them all. However, in January 1998 the company filed for Chapter Eleven bankruptcy protection, after finding itself unable to pay its bills.

Ⅳ. Conclusion

The theory of auctions is still poorly developed even though the use of auctions is prevalent in market transactions from the ancient era to the present. The obvious reason for not achieving a satisfactory theory of bidding is the tremendously complex economic environments in which auctions are conducted. The celebrated revenue equivalence result has been obtained by solving the problem of designing auctions to maximize the seller's expected revenue (Harris and Raviv (1981), Myerson (1981), Riley and Samuelson (1981)). But this equivalence result breaks down 1) when either the seller or the buyers are risk averse (Harris and Raviv (1981), Holt(1908), Maskin and Riley (1980), and Matthews (1979)), in that case Dutch or first price auctions are preferred to the English or second price auctions; 2) when the valuations of the object of the object being sold are affiliated (Milgrom and Wever (1982)), in that case, the English(ascending) auction generates higher average prices than does the second price auctions.

Several extensions have been obtained by introducing collusions among bidders (McAfee and Reny (1982)), allowing resale market after auctioning the object Zheng (2002), and financial constraint in bidding (Che and Gale (1998)). But some issues about auctions for shares of a divisible object and simultaneous auctions of several hundred objects of different value are not satisfactorily resolved.

Auction theory is a central part of economics, and should be a part of every economist's armory; auction theorists' ways of thinking shed light on a whole range of economic topics.

This paper is short for practical auction and case study. We leave for future research.

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