濟州大學校 尖端技術硏究所 論文集, 第14卷, 2號, pp.236-239, 2003. 12
Cheju Nat'l Univ. Res. Inst. Adv. Tech. Jour., Vol.14, No.2, pp.236-239, 2003. 12

Adaptive Mesh Regeneration 기법을 이용한 전기 임피던스 단층 촬영술에서의 경계 추정

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Estimation of Phase Boundaries in Electrical Impedance Tomography by Adaptive Mesh Regeneration Technique

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ABSTRACT

In electrical impedance tomography(EIT) the impedance distribution, that is the phase distribution is usually reconstructed in fixed elements inside the system. In practical cases, such as the impedance imaging of two-phase flow, this model might not be useful. For the two-phase flow system, the impedance of each phase doesn't change but instead the phase boundary depends on the distribution of dispersed phase. In the present study, an new image reconstruction algorithm employing adaptive mesh regeneration technique is developed for the detection of phase boundary. The feasibility of this method is tested for some numerical simulations and phantom experiments.

Key Words : Electrical Impedance Tomography, Boundary Estimation, Adaptive Mesh Regeneration

Ⅰ. Introduction

Two-phase flow can occur under the normal and accidental conditions in various processes such as heat exchanger, steam power generation and oil or natural gas pumping system. Because the phase distribution affects the safety, control, operation and optimization of process, it is important to know the phase boundaries in on-line without disturbing the flow field. Recently, the electrical tomography technique is employed to investigate two-phase flow phenomena, because it is relatively inexpensive and has good time resolution.

The poor spatial resolution of EIT, which is due to the diffusive characteristics this method, the exact phase boundary is hardly reconstructed. This problem is major drawback in the application of

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EIT into the real worlds. However, EIT is sensitive to relatively small impedance changes and the data needed for the image reconstruction can be acquired and analyzed fast, which makes it possible to visualize phase boundary in two-phase flow.

In the present study, an new image reconstruction algorithm employing adaptive mesh regeneration technique was developed for the detection of phase boundaries. The phase boundaries were expressed as truncated Fourier series and the Fourier coefficients were estimated with the aid of finite element calculation. To test the feasibility of the method, some numerical simulations and phantom experiments were conducted.

II. Electrical Impedance Tomography

The EIT was originally developed for the medical tomography. Mathematically, the EIT is composed of the forward problem to obtain the voltage distribution subject to assumed impedance distribution and the inverse problem to reconstruct the impedance distribution under the measured boundary voltages. The details of the forward and inverse problems are discussed as below.

2.1. Forward Problem

When the impedance(resistivity) distribution ρ and boundary current I_l are given, the electrical potential distribution u within the problem domain Ω is governed by the following Laplace equation and the Neuman type boundary conditions

$$\nabla \cdot \left(\frac{1}{\rho} \nabla u\right)=0, \quad x\in\Omega \tag{1}$$

$$u+z_\ell \frac{1}{\rho}\frac{\partial u}{\partial \nu}=U_\ell \quad x\in e_\ell, \quad \ell=1,2,\cdots,L \tag{2}$$

$$\int_{e_\ell} \frac{1}{\rho}\frac{\partial u}{\partial \nu}dS=I_\ell, \quad x\in e_\ell, \quad \ell=1,2,\cdots,L \tag{3}$$

$$\frac{1}{\rho}\frac{\partial u}{\partial \nu}=0, \quad x\in\partial\Omega/\cup_{l=1}^{L} e_\ell \tag{4}$$

where e_ℓ is ℓ'th electrode, z_ℓ is effective contact impedance between ℓ'th electrode and the object, U_ℓ is the voltage on the ℓ'th electrode, ν is the outward directed normal vector. In addition, the following two conditions for the injected current and measured voltages are needed to ensure the uniqueness of the solution.

$$\sum_{l=1}^{L} I_l=0 \tag{5}$$

$$\sum_{l=1}^{L} U_l=0 \tag{6}$$

Since the above equation can't be solved analytically for the arbitrary impedance distribution, the numerical method such as FEM method should be employed to obtain the solutions. In most of EIT problem, the impedance within the element assumed to be constant, then the above differential equation is approximated by the system of algebraic equations. The details of the solution method are given in Vauhkonen's work1).

2.2. Inverse Problem

The inverse problem of EIT maps the boundary voltages from real or artificial experiments to impedance image. The objective function may be chosen to minimize the square error,

$$\Phi=[V-U]^T[V-U] \tag{7}$$

where V is the vector of measured voltage and U is the calculated boundary voltage vector that must be matched to V.

Because of the mismatch between the FEM mesh structure and the real phase distribution, the blurred image is unavoidable. The basic idea of the proposed approach is following. If we known the phase boundary, we can generate FEM mesh where is no element which lie on both phase. The phase boundaries $\{C_l, l=1,2\cdots S\}$ can be approximated in the form

$$C_l(s)=\begin{pmatrix} x_l(s) \\ y_l(s) \end{pmatrix}=\sum_{n=1}^{N_\theta}\begin{pmatrix} \gamma_n^{x_l}\theta_n^x(s) \\ \gamma_n^{y_l}\theta_n^y(s) \end{pmatrix}, \quad l=1,\cdots,S \tag{8}$$

where $\theta_n(s)$ are periodic and smooth basis function. In this study we express both coordinates of the curve as Fourier series with respect to the curve parameter s, that is, we use basis function of the form

$$\begin{aligned} &\theta_1^\alpha(s)=1 \\ &\theta_n^\alpha(s)=\sin(2n\pi s), \quad n=2,3,\ldots \\ &\theta_n^\alpha(s)=\cos(2n\pi s), \quad n=2,3,\ldots \end{aligned} \tag{9}$$

where $s\in[0,1]$, and α denotes either x or y. Furthermore, using the expansion of (8), the boundaries $\{C_l\}$ are identified with the vector γ of the shape coefficients, that is,

$$\gamma=(\gamma_1^{x_1},\ldots\gamma_{N_\theta}^{x_1},\gamma_1^{y_1},\ldots\gamma_{N_\theta}^{y_1},\ldots\gamma_1^{x_S},\ldots\gamma_{N_\theta}^{x_S},\gamma_1^{y_S},\ldots\gamma_{N_\theta}^{y_S}) \tag{10}$$

where thus $\gamma\in Re^{2SN_\theta}$. In present study, for the simplicity of analysis we set $S=1$ and $N_\theta=2$. Based on the coordinates of phase boundaries, we generate FEM mesh structure where no element cross the phase boundaries.

The mapping F from the coefficients γ's to the measured potential is highly nonlinear. When the impedance of both phases are assumed to be known, the impedance information is implicit in the mapping F. We linearized the mapping $F:\gamma\rightarrow U$ at some point (ρ_*, γ_*) to obtain

$$U=U_*+J_F(\gamma-\gamma_*) \tag{11}$$

where U_* are the measured potentials that correspond to (ρ_*, γ_*).

The Jacobian is calculated as follows. A FEM mesh that corresponds to the coefficients γ_* is constructed. The columns J_F^m of J_F are then obtained by perturbing each of the coefficients γ's by $\delta\gamma$, regenerating FEM mesh so that it again osculate with the perturbed boundary and calculating the resulting potentials U' on the system boundary. We have then

$$J_F^m=(\delta\gamma)^{-1}(U'-U_*) \tag{12}$$

III. Numerical Simulation

We assumed phase distribution and calculated the boundary potentials. Based on these potentials, the proposed method was tested. In Fig. 1, we consider an artificial single object located at an arbitrary position. Although it seems to be quite simple, this example is a very illustrative example. As shown in this figure, the proposed method works well. The conductivity contrast ratio between dispersed phase and continuous phase is assumed to be 1000 for both cases.

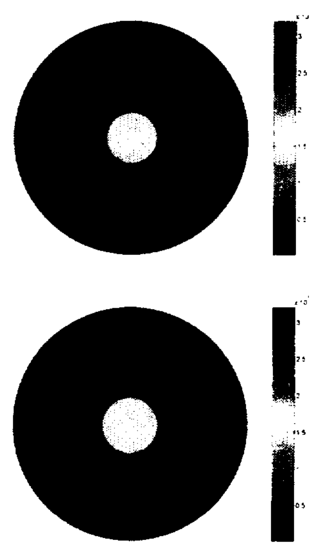


Fig. 1. Reconstructed results; true(assumed) images (top) and reconstructed images(bottom).

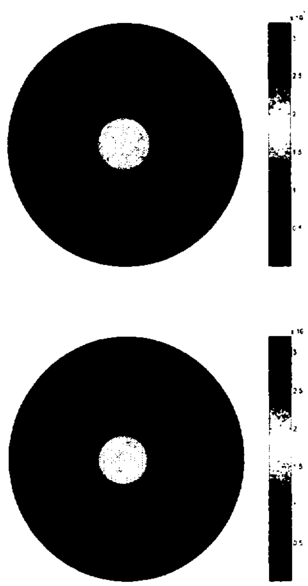
Ⅳ. Image Reconstruction Based on the Phantom Experiments

We carried out measurements in a cylindrical phantom with saline solution and nonconducting plastic rod, as shown in Fig. 2. The geometry is two-dimensional and the current generated by current generation circuit was injected into the 32 electrodes in the following form

$$I_l^k = \begin{cases} \cos(k\zeta_l) & k = 1, 2, \cdots, 16 \\ \sin(k\zeta_l) & k = 1, 2, \cdots, 15 \end{cases} \qquad (13)$$

where $l = 1, 2, \cdots, 32$, and $\zeta_l = l/32$, and the resulting potentials were measured simultaneously.

Based on the boundary potentials measured experimentally, the phase boundary is reconstructed with the proposed method and conventional modified Newton-Raphson method2). As shown in Fig. 2, the proposed method results in much better images than the conventional method. So, it seems that the proposed method can be used to estimate the phase boundary.



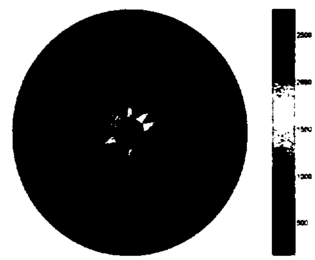


Fig. 2. Reconstructed results: true images(top), images reconstructed by proposed method (middle), images reconstructed by conventional Newton-Raphson method(bottom).

Ⅴ. Conclusion

The algorithmic and experimental study were conducted to phase boundary by EIT. The reconstructed images based on our experimental results show that the phase boundary can be identified by the proposed method. It is expected that the EIT can be used to monitor various process systems where the two-phase and/or two-component transport exists.

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