

히다닉 모델의 확인문제

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The Problem of Identification in Hedonic Model

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본 논문은 히다닉(Hedonic) 모델의 확인(Identification)문제를 분석하였다. 만약 히다닉 가격함수가 비선형이라면 각각의 특성에 대해 가격변동이 있다. 그래서 그러한 특성들이 함께 선택되어지기 때문에 암시적인 특성들의 가격과 양은 내생변수가 된다. 이것이 우리가 얘기하는 소위 “연립방정식 편위”이다.

이러한 문제를 해결하기 위해 Mendelsohn은 수요함수를 추정할 때 추정된 한계가격을 사용할 것을 제안하였다. 즉, 외생적인 한계가격 변동의 필요성을 주장하였다.

더욱이 단일시장에서는 수요함수를 확인하기 위해 히다닉 방정식의 함수형태를 제한할 필요가 있다. Diamond와 Smith는 히다닉 모델에서 확인문제를 해결하기 위한 방법으로 시장을 분리할 것을 제안하였다. 그렇게 하면 수요추정의 다음 단계로 이동이 가능하다고 주장하였다.

주택시장의 경우 각각의 특성에 대한 요구들 사이에 가능한 상호작용을 설명하기 위해 주택의 속성들에 대한 수요체제가 구체적으로 명시되어야 한다. 위의 조건이 만족될 경우 실질적인 수요분석을 위해 융통성 있는 함수 형태인 LA/AIDS 모델을 사용함으로써 히다닉 모델의 확인 문제를 해결할 수 있다.

I. Introduction

For many years, hedonic price techniques have estimated the impacts of various characteristics on the price of a housing good. The hedonic approach has been used to estimate the demand for a characteristic in a heterogeneous housing product. For instance, impacts of public goods such as air quality, crime rate, and neighborhood quality have been analyzed by this approach.

Rosen(1974) provided a widely used model, the two-step approach, in which there exist demand and supply functions for the various characteristics of a differentiated product. Following the technique of Rosen, most research on the housing market has estimated the underlying demand and supply functions for various characteristics such as the examples above. However, Rosen's two step technique has been criticized theoretically and empirically by many researchers because it does not adequately address the problem of identifying the demand equation for attributes.

The paper will present Rosen's theoretical and

empirical methods, the identification problem, market segmentation as a solution to the identification problem, and the complete demand system.

II. Rosen's Model

1. Rosen's Theoretical Model

A general model of implicit markets for attributes in a competitive market equilibrium in a single market was developed by Rosen. Suppose that buyers know that a unit of a good will bring different levels of utility by having different sets of attributes. Sellers know that different sets of attributes have different costs.

A household's willingness to pay for a unit with more of a particular attribute is revealed by the household's bid function. In mathematical notation, the bid function is written as, $\theta(Z, U, Y_1)$ where Z is a vector of attributes of the good, $U = U(Y - \theta, Z)$, Y is income, and Y_1 is a vector of demand shifters such as socio-economic factors.

A producer's willingness to accept the good with more of the attribute is explained by producer's offer function. Notationally, the offer function is written as $\Phi(Z; \pi, Y_2)$ where

π is a constant profit level and Y_2 is a vector of supply shifters such as labor cost. Rosen defined the hedonic price equation as a locus of market equilibrium points of various quantities of the attribute. The hedonic price equation, $P(Z)$, which is an envelope of bid and offer functions in equilibrium is pictured in Figure 1(a).

The first derivative of the bid function $\theta(Z, U, Y_1)$ with respect to a particular attribute Z_j is the marginal bid function, θ_j , which implies the marginal willingness to pay for Z_j . The first derivative of the offer function $\Phi(Z; \pi, Y_2)$ with respect to Z_j gives a marginal offer function, Φ_j , which is the marginal willingness to accept for Z_j . At market equilibrium the marginal bid price for Z_j equals the marginal offer price for Z_j (see Figure 1(b)). The locus of equilibrium points of the marginal bid and the marginal offer functions defines an implicit marginal price function. Thus, utility is maximized when the following conditions hold:

- (1) $\theta(Z^*, U^*, Y_1) = P(Z^*)$
 $= \Phi(Z^*, \pi^*, Y_2)$
- (2) $\theta_j(Z^*, U^*, Y_1) = P_j(Z^*)$
 $= \Phi_j(Z^*, \pi^*, Y_2)$

for all i , where $*$ indicates optimal levels.

As shown in Figures 1(a) and 1(b), variation of the demand shifter, Y_1 , will move a household from one optimum to another. Likewise, variation of the supply shifter, Y_2 , will move a producer from one optimum to another.

2. Rosen's Empirical Method

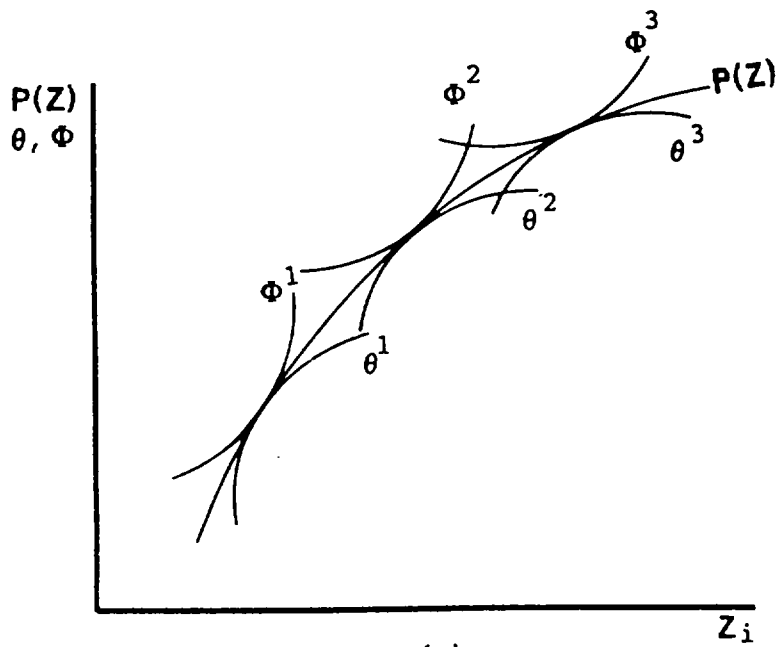
The traditional technique for demand estimation has been the two-step method proposed by Rosen. The method begins with the estimation of the hedonic price function on all of the various characteristics. From the hedonic price function, the implicit prices of characteristics are derived. In the second step, marginal prices of the characteristics for each property are used as the dependent variables in which demand and supply functions for characteristics are simultaneously estimated.

Rosen's empirical model to be estimated will be as:

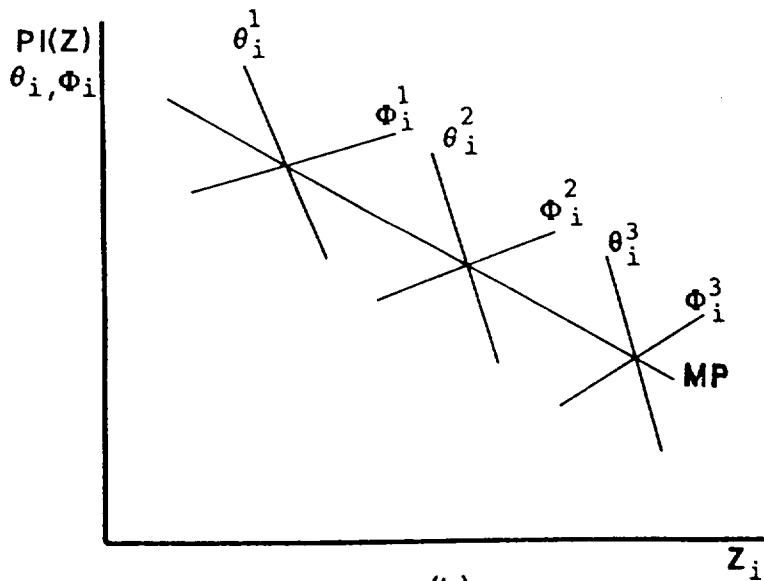
$$(3) P_i(Z) = \theta_i(Z, Y_1) + e_1 \text{ [Demand]}$$

$$(4) P_i(Z) = \Phi_i(Z, Y_1) + e_2 \text{ [Supply]}$$

for all $i=1, \dots, n$, where Y_1 and Y_2 are exogenous demand and supply shift variables, respectively, and $e_i, i=1,2$, are error terms. Estimation of this model requires such a two



(a)



(b)

Figure 1. Market Equilibrium in an Implicit Market

step procedure. Rosen claimed that the two step approach employed for the estimation leads to the "garden variety" simultaneity of demand and supply.

Rosen also argues that hedonic demand and supply functions can be treated as the traditional demand and supply functions for goods. Thus, using a two-step procedure one can estimate the demand and supply functions with the data from a single market as long as the marginal prices of attributes are not fixed. Additionally, Rosen asserts that if an individual wishes to value nonmarginal changes in characteristics, this can be done in a straightforward manner. This is because hedonic demand and supply functions are equivalent to the traditional demand and supply functions for goods.

Simultaneous estimation of the demand and supply functions for housing attributes in the single market has been done using Rosen's two step approach. If the hedonic price function is linear, then estimated parameters in the first step cannot be used to estimate demand functions in the second step as there is no price variation for attributes.

In other words, without price variation for attributes in a single market, we cannot identify the parameters of a demand function. If the hedonic price function estimated in the first step is nonlinear, then there is price variation for characteristics. However, both implicit prices and quantities of attributes are endogenous variables since they are jointly chosen. In fact, price variation in a single market will always be endogenous.

Thus, in either case, Rosen's approach cannot be used to estimate the consistent parameters of the demand functions for attributes (Parsons, 1984).

However, researchers (Brown and Rosen, 1982; Diamond and Smith; Mendelsohn, 1984 and 1986) began to realize that the feasibility of second stage of Rosen's two step approach depends upon severe restrictions. For the single market hedonic models, Mendelsohn (1986) stated that there are three potential problems to identify the structural equations in a single market: (1) the simultaneity of supply and demand equations, (2) the use of predicted marginal prices in the structural equations for

identification, and (3) the variability of prices and exogenous structural shift variables throughout the sample. But Mendelsohn did not exclude the possibility that hedonic structural equations could be estimated with segmented market data.

III. The Problem of Identification

1. The Origin of the Identification Problem

A successful empirical estimation of the demand for housing characteristics in an implicit market requires a thorough understanding of the nature of the identification of demand and supply equations. Intuitively, identification is a problem of model formulation. A model is identified if it is in a unique statistical form. In order to determine that the sample data identify the demand or supply functions, it is necessary to know the changes in the other factors which impact the supply and demand functions (Koutsoyiannis, 1977).

Koutsoyiannis states that if each equation in a structural system of equations contains

the same explanatory variables, the model cannot be identified and is statistically impossible to estimate. There are many factors other than price which can influence demand and supply schedules. Changes in these exogeneous factors leads to shifts of the supply and demand curves.

Observable price and quantity combinations in an explicit market represent the points corresponding to particular demand and supply schedules (Ohsfeldt, 1983). In this case, as mentioned in the above, the changes in exogenous factors affecting demand and supply curves allow identification of demand and supply functions. In other words, if we know the factors that cause the shifts, then we may be able to identify both functions, or one of them. For example, if a demand schedule shifts because of income changes and a supply schedule shifts because of weather conditions, then one has the model $D=F(P,Y)$, where Y is income, and $S=G(P,W)$, where W represents weather, and both demand and supply functions can be identified.

However, Ohsfeldt (1983) states that in the case of implicit markets, "point

estimates of marginal prices can only be obtained by using information on the quantities of characteristics observed." In addition, a change in an exogenous factor affecting demand is simultaneously determined with a change in an exogenous supply factor since no individual wants to buy (or sell) more than one combination (Ohsfeldt, 1983). Thus, the marginal bid function, θ_i , and the marginal offer function, Φ_i , cannot be identified since Y_1 and Y_2 in equations (3) and (4) vary jointly. Thus, the simultaneity problem due to the presence of nonlinearity in the hedonic market becomes a very important issue for housing characteristics.

Diamond and Smith (1985) state that the simultaneity arises in the markets due to nonlinearity of the hedonic price function, not due to the interactions of supply and demand schedules. Intuitively, nonlinearity in the demand side means that marginal prices of attributes for a household are functions which are the choice of which housing bundle to consume. In the supply side, it implies that exogenous marginal prices of a supplier are functions of

his choice of which housing bundle to produce. Therefore, exogenous marginal price variation will be needed to identify the demand function for a housing characteristic. Such marginal price variation is observable in the case of single market data, although with strong a priori restrictions on functional forms (Diamond and Smith). However, such strong a priori restrictions are not necessary with segmented market data. Thus, Diamond and Smith suggest the use of market segmentation to estimate household demand for a characteristic.

2. Market Segmentation

Several researchers (Palmquist, 1984; Brown and Mendelsohn, 1984; Parsons, 1984, King, 1976; Ohsfeldt, 1983; Kim, 1992) have used market segmentation to properly estimate hedonic structural equations. They assume that there exist many markets for a heterogeneous housing products and that consumers with identical preferences may be observed across markets. Price variation in the different markets is exogenous under this assumption. Thus, demand functions can be consistently

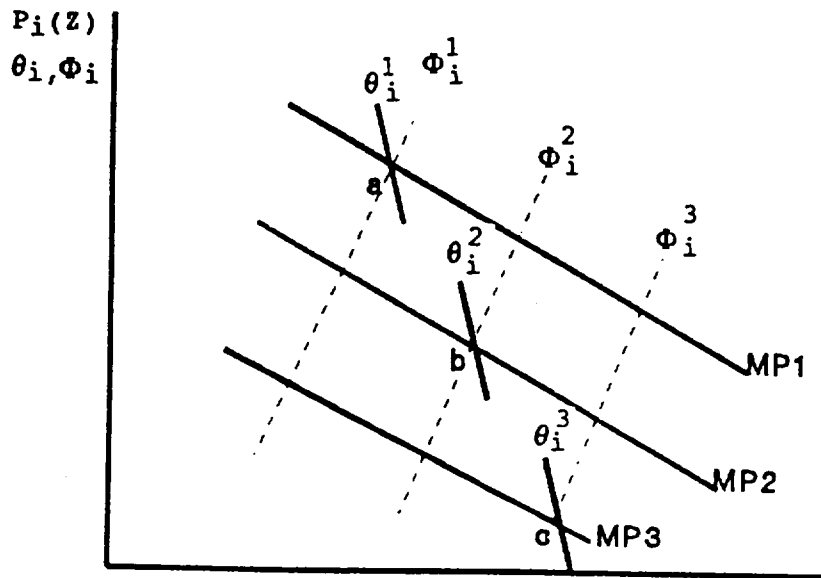
estimated, regardless of whether hedonic price functions are linear or nonlinear.

In this regard, these researchers have concluded that the use of segmented market data holds the most promise for obtaining preference parameters for the structural equations. Follain and Jimenez (1985) comment that "the ideal data set is a multiple market data set that includes variables on individual housing units...." McConnell and Phipps (1986) argue that market segmentation might exist in housing market in different cities or possibly in different areas of the same city. However, Mendelsohn (1986) cautioned that developing segmented markets by arbitrarily subdividing a single market into submarkets is not appropriate.

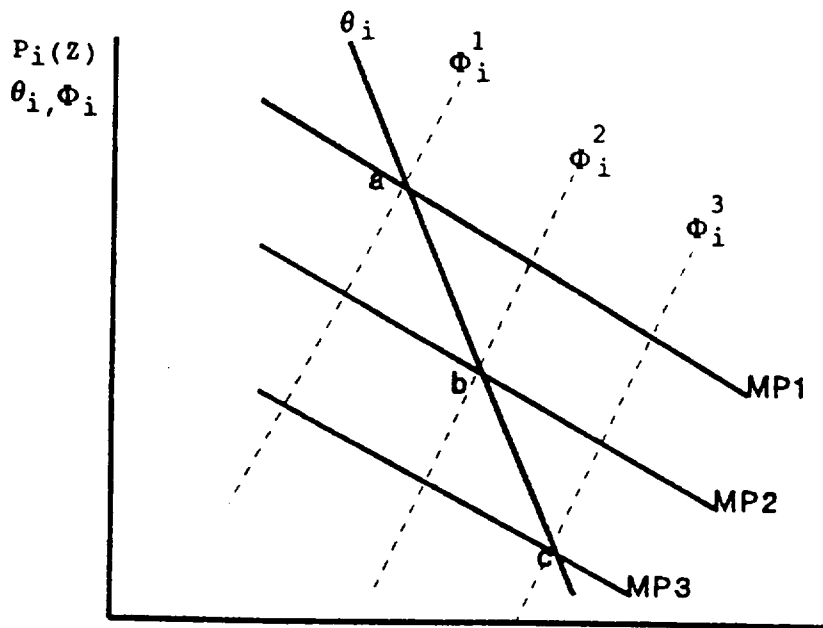
The technique suggested by Brown and Rosen and by Brown and Mendelsohn employs data from segmented markets so that separate hedonic equations can be estimated for each distinct market. Then, the MP_i (marginal price function i), where $i=1, 2, \text{ and } 3$, is derived from each hedonic price equation. As shown in Figure 2(a) and 2(b) if consumers

within one market have identical preferences, the point "a" is chosen in market 1 by the representative of this market, the point "b" in market 2, and the point "c" in market 3.

Further assume that multimarket representatives have identical preferences. Then the variation in the marginal prices from distinct markets and the shift of demand curve due to demographic differences allow identification of the demand function (See Figure 2(b)). In this case, the hedonic price function need not be restricted to be nonlinear for the identifiability of the demand equation. However, if multimarket representatives have different preferences, there is no way of identifying the demand equation (See Figure 2(a)). Therefore, the key points of identification are the variation in the marginal prices from distinct markets, the assumption of identical preferences across markets, and demographic differences. These assumptions are commonly made in empirical hedonic studies.



(a) Different Preferences



(b) Identical Preferences

Figure 2. Identification of Demand Function with Market Segmentation

IV. Estimation of a Complete Demand System

1. Complete Demand System

Many researchers (Blomquist and Worley, 1982; Palmquist, 1984; King, 1988) have estimated only a demand function of one or a few characteristics at a time, rather than a complete demand system. According to Loehman and Tolley (1989) the level chosen for one characteristic affects the levels chosen for others. For instance, a household may choose a smaller house in a safe area with a large setback distance. Thus, they argue that "a complete demand system approach, rather than a single demand equation approach, should be used to obtain an appropriate model."

There are several possible specifications of a demand system. They are the linear expenditure system (LES), the direct and indirect translog, the quadratic expenditure system, the S-branch, the Laurent, the generalized Leontief, and the almost ideal demand system (AIDS). The AIDS was first developed and estimated by

Deaton and Muellbauer (1980). Since then many researchers (Blanciforti, Green, and King, 1986, Chern, et al., 1988, Earles and Unnevehr, 1988, and Kim, 1992) have used the linear approximate almost ideal demand system (LA/AIDS) in empirical analysis as a good first-order approximation to the full AIDS model.

One advantage of using the AIDS model (a complete demand system) is that it allows for the testing of the theoretical restrictions such as adding up, homogeneity, symmetry, and negativity (Blanciforti, et al., 1986). Another advantage is that it allows price and income elasticities to vary over time. Since Deaton and Muellbauer (1980) introduced AIDS, it has been widely used to estimate demand for consumer goods but not the attributes of housing. Parsons (1984) is the only one who used a demand system approach with the AIDS system and multi-market data in order to get variation in the implicit prices of characteristics across markets. Exact measures were obtained for nonmarginal changes in neighborhood characteristics. However, the imposition of a prior restriction on the

functional form for the linear hedonic price equation is a shortcoming of his hedonic regressions. Thus, Kim(1992) has estimated a complete demand system with nonlinear hedonic price equations and segmented market data. That research among the first to use a complete demand system with nonlinear hedonic model and segmented market data

2. Specification of Demand Model

Assume that the characteristics in each market are given exogenously and allocated among consumers. Precisely, the supply of attributes is exogeneous. Also assume that the utility function depends only on the bundle of differing quantities of characteristics, Z_i . Thus, the household derives utility from consuming a given bundle of characteristics with the utility function which depends on a vector of characteristics. This is possible when the utility function is assumed to be additively separable. Then, households maximize utility given the utility function and a budget constraint. In functional form, the consumer chooses z so as to maximize:

$$(5) U=U(Z)$$

$$(6) \text{ s.t. } Y_h=P(Z)$$

where

Z : the bundle of attributes

P : the hedonic price function

Y_h : budget to spend for purchasing the house

Palmquist (1984) states that the use of traditional demand theory and estimation techniques requires the linearization of the budget constraint. Normally, the budget constraint in the consumer optimization problem is linear. In implicit markets, however, the budget constraint is nonlinear because the hedonic price model is nonlinear. Thus, it is necessary to linearize the budget constraint to avoid the excessive nonlinearity. As long as the budget contour is concave to the origin, the utility maximization problem with a nonlinear budget constraint is equivalent to utility maximization with a linear budget constraint since the budget constraint is tangent to the indifference curve at the optimum bundle (Mendelsohn, 1986). Further, the linearized budget constraint at the optimum bundle is tangent to the indifference curve at the same point.

Palmquist also argues that altering the constraint to a

linear form requires the adjustment of the expenditure variable so that the linearized budget constraint would still be tangent at the perceived consumption bundle. Following Palmquist, add $\sum P_i Z_i$ to both sides of equation (6), and then subtract $P(Z)$ from each side of equation.

Then, the equation (6) will be as follows:

$$(6)' Y_h^* = Y_h + \sum P_i Z_i - P(Z) \\ = \sum P_i Z_i$$

where Y_h^* is the adjusted income and P_i is the marginal price of characteristic Z_i . In order to avoid biased estimates of demand equations this adjustment procedure is necessary.

Now the above consumer optimization problem theoretically could be solved for a system of demands:

$$(7) Z_1 = Z_1(Y_h^*, D_t, P_i); i=1,2,\dots,n \\ Z_2 = Z_2(Y_h^*, D_t, P_i); i=1,2,\dots,n \\ \cdot \\ \cdot \\ \cdot \\ Z_n = Z_n(Y_h^*, D_t, P_i); i=1,2,\dots,n$$

$$\text{where } P_i = \frac{\partial P}{\partial Z_i}$$

Equation (7) implies that the quantity demanded of trait i depends on its marginal price, P_i , adjusted household income, Y_h^* , and socioeconomic factors,

D_t . Freeman(1979) argued that the second stage hedonic technique combines the quantity and implicit price information in an effort to identify the inverse demand function.

To estimate the demand for any attribute, the partial derivatives of the hedonic equations with respect to the attributes have to be estimated to provide marginal prices for the characteristic demand functions. Thus, marginal prices for attributes, $\partial P_{ijk}(Z)/\partial Z_{ijk} = PZ_{ijk}$, are initially estimated for each observation and for each endogenous trait (PZ_{ijk} is the j th hedonic price for unit i in submarket k). In the first stage of two stage least squares (TSLS) technique, the estimated prices from the hedonic regressions are used as dependent variables to regress on all demand shifters which are all the exogenous variables (Mendelsohn, 1984), that is:

$$(8) P_i = F(I, C_i); i=1,2,3$$

where I is household income and C_i are county dummy variables. In the second stage of TSLS, these predicted marginal prices are used as independent variables to estimate a demand function for each trait along with

expenditure and demographic variables. In this procedure, it is necessary to assume that each individual determines exogenous prices for each trait.

In the next section, the AIDS is described and much of the discussion depends on Deaton and Muellbauer (1980) and Blanciforti, et al. (1986).

3. Specification of The LA/AIDS For Housing Attributes

The derivation of the AIDS is from an underlying structure of consumer preferences via a cost or expenditure function of the form:

$$\log c(u,p) = a_0 + \sum_k a_k \log p_k + \frac{1}{2} \sum_k \sum_j b_{kj}^* \log p_k \log p_j + u d_0 \pi p_k^{d_k}$$

where u is utility, p is a price vector, and a_0 , a_i , b_{kj} , and d_j are parameters. Using Shephard's Lemma, the demand functions in budget share form can be derived by differentiating the cost function and appropriate substitutions. Then, the budget share can be written as:

$$W_i = a_i + \sum b_{ij} \log p_j + c_i \log(Y/P)$$

where P is a price index defined as:

$$\log P = a_0 + \sum a_k \log p_k + \frac{1}{2} \sum_k \sum_j b_{kj}^* \log p_k \log p_j.$$

Since this original AIDS model is highly nonlinear, it is often difficult to estimate the parameters. Thus, it is common to use Stone's Price index (P^*) instead of a general price index (P) for a linear approximation of the demand system. The Stone's price index is defined as:

$$\log P^* = \sum W_{kt} \log P_{kt}.$$

The use of the Stone's index has been shown to have a little effect on the value of the log likelihood function (Anderson and Blundell, 1983). Deaton and Muellbauer (1980) state that the model that uses Stone's price index is called "the linear approximate almost ideal demand system" (LA/AIDS). Then, the LA/AIDS can be written as:

$$W_i = a_i + \sum b_{ij} \log p_j + c_i \log(Y/P^*).$$

They also mention that the LA/AIDS is "a good first-order approximation to the complete AIDS system". Deaton and Muellbauer (1980) also note that the linear approximation in most cases is fairly close to the AIDS model. An advantage of using LA/AIDS instead of AIDS is that it is easier to estimate. Further, for LA/AIDS as for AIDS, the demand properties, such as adding up,

homogeneity, and symmetry, can be easily shown to be satisfied.

In order to incorporate the impacts of demographic variables into the system, the procedure developed by Pollak and Wales (1980, 1981) is used. Demographic variables such as age, sex, family size, and level of education are denoted by $D=(d_1, d_2, \dots, d_n)$. Pollak and Wales argue that these demographic variables are major determinants of household consumption patterns. Jeng and Hushak state that "these demographic variables affect the individual's preferences, and hence the parameters of the individual's utility function are functions of the d_i 's." In this section, it is hypothesized that socioeconomic factors are incorporated into an intercept shift of the demand function. Thus, the demand function associated with the parameters of the demand function itself is:

$$(9) \quad Z_{it}^d = Z_{it}^d [P_1, \dots, P_n; \alpha, Y; \beta, \theta (D)]$$

where P_i denotes marginal prices, Y is total expenditure, and θ is the parameter vector (i.e., α, β) of marginal prices and household income and is

also a function of D . Pollak and Wales developed five general procedures for incorporating socioeconomic variables into the demand system. Demographic translating is one of five procedures which can be tested to determine whether the demographic variables are incorporated into intercept shifts of the demand function. Pollak and Wales show that linear demographic translating is as follows:

$$(10) \quad D^i(d) = \sum_{r=1}^n \delta_{ir} d_r.$$

Equation (10) implies that the demand function varies in intercept when incorporating the demographic variables into the system. However, if the demographic variables have no effects on the demand system, then $\delta_{ir}=0$.

The linear demographic translation is added to the original demand system. Then, the empirical model to be estimated is:

$$(11) \quad W_i^* = a_i + \sum \delta_{ij} d_j + \sum b_{ij} \log p_j + c_i \log (Y/P^*)$$

where $\log P^* = \sum W_k \log P_k$, Stone's (1953) index, Y is total expenditure, and $W_i = P_i Z_i / Y$.

If the demographic variables have no effects on the demand

system, then δ_{ij} is zero. Then the LA/AIDS model is the same as the original LA/AIDS model.

Conventional demand properties, such as adding-up, homogeneity, and symmetry, need to be tested. The adding-up condition requires that the budget shares sum to 1 if:

$$\sum_i a_i = 1, \sum_i b_{ij} = 0, \sum_i c_i = 0,$$

and $\sum_i \delta_{ij} = 0.$

The property of homogeneity (i.e., homogeneous of degree zero in budgets) demands the following condition:

$$\sum_i b_{ij} = 0; \quad \text{for all } i.$$

The symmetry condition holds if:

$$b_{ij} = b_{ji}; \quad \text{for all } i \text{ and } j.$$

In the LA/AIDS model, the adding up condition is a maintained hypothesis although the other restrictions are testable even though these restrictions may or may not be imposed in the model. However, a complete demand system satisfying the above properties is called an admissible demand system.

V. Summary

The paper analyzed the identification problem. If the hedonic price function is nonlinear, then there is a price variation for characteristics. Thus, both implicit prices and quantities of attributes become endogenous variables since they are jointly chosen. This is what we called the simultaneous equation bias. In order to solve this problem, Mendelsohn suggested using predicted marginal prices for demand estimation. In other words, exogenous marginal price variation is needed.

Furthermore, in a single market it is necessary to restrict the functional form of the hedonic equation to identify the demand equation. Diamond and Smith suggested the use of market segmentation as a solution of the identification problem. Thus, market segmentation will allow the next step of demand estimation. To account for possible interactions among the demands for characteristics, a demand system for housing characteristics is specified. A flexible functional form, the LA/AIDS, is used for empirical demand analysis.

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