濟州大學校 尖端技術硏究所 論文集, 第14卷, 2號, pp.144-150, 2003. 12
Cheju Nat'l Univ. Res. Inst. Adv. Tech. Jour., Vol.14, No.2, pp.144-150. 2003. 12

이동로봇의 Fuzzy 모델링 및 제어

강 진 식*

Fuzzy Modeling and Control of Wheeled Mobile Robot

Jin-Shig Kang*

ABSTRACT

In this paper, the control of the differential drive wheeled mobile robot (DDWMR) is studied. Because the DDWMR have non-holonomic constraints, it cannot be stabilized by smooth feedback. The T-S fuzzy model for the DDWMR is presented and a control algorithm is developed by well known PID control and LMI based regional pole-placement.

Key Words : T-S Fuzzy Model, Wheeled Mobile Robot, LMI, Pole-Placement

Ⅰ. INTRODUCTION

Various kinds of mobile robots have been developed and recently, many of scalars devote their efforts on these areas. Among them, the differential drive and the car like wheeled mobile robot(WMR) are most widely used in their application field. Since the WMR system is a typical non-holonomic system except the omni-directional types, the standard control laws must be developed for systems with non-holonomic constraints. Due to the fatal property that a WMR with nonholonomic constraints cannot be stabilizes by a smooth feedback, it is necessary to find more effective and advanced algorithms[1-3].

In paper, the control of the differential drive wheeled mobile robot(DDWMR) is studied. Because the DDWMR have non-holonomic constraints, it cannot be stabilized by smooth feedback. The T-S fuzzy model for the DDWMR is presented and a control algorithm is developed by well known PID control and LMI based regional pole-placement.

Ⅱ. Modeling of Wheeled Mobile Robot

2.1. Dynamic Modeling of Wheeled Mobile Robot [1, 4]

The structure of the mobile robot, considered in this paper, is shown in Fig. 1. The relation between the forward velocity and the wheel angular velocity is described by

* 제주대학교 통신컴퓨터공학부
Faculty of Telecommunication and Computer eng., Cheju Nat'l Univ.

$$\begin{bmatrix} v \\ \dot{\varnothing} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & -\frac{r}{2b} \end{bmatrix} \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} \quad (1)$$

where, v and $\dot{\varnothing}$ are forward and rotation velocities of the robot, respectively, and r is the ratio of the wheel. And b is the displacement from center robot to center of wheel. The kinetic equation is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varnothing} \end{bmatrix} = \begin{bmatrix} \cos\varnothing & 0 \\ \sin\varnothing & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \dot{\varnothing} \end{bmatrix} \quad (2)$$

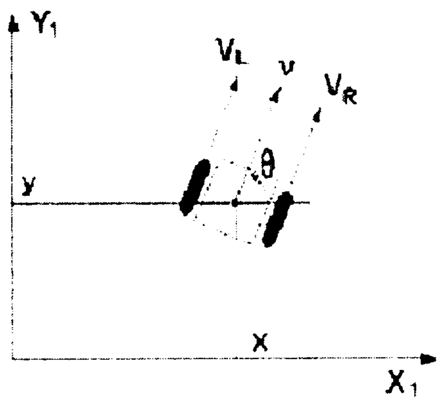

Fig. 1. Structure of WMR

In order to derive the dynamic equations, we now define some variables.

Ic : robot inertia except wheels and rotor

Iw : motor rotor inertia for wheels and wheel axis

Im : motor rotor inertia for wheels and wheel diameter

m : mass of robot except wheels and motor rotor

mc : mass of wheels and motor rotor

The dynamic equation of a of robot is described by[4,5]

$$M(q)\ddot{q} + V(q, \dot{q}) = E(q)\tau - A^T(q)\lambda \quad (3)$$

where, λ is Lagrangy multiplier, τ is the torque of each wheels, and d is the displacement from the center of mass to the center of rotation, $q = [x \ \ y \ \ \dot{\theta}_r \ \ \dot{\theta}_l]^T$ and

$$A(q) = \begin{bmatrix} -\sin\varnothing & \cos\varnothing & 0 & 0 \\ -\cos\varnothing & -\sin\varnothing & cb & cb \end{bmatrix}, \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

$$M(q) = \begin{vmatrix} m & 0 & -m_c cd\sin\varnothing & m_c cd\sin\varnothing \\ 0 & m & m_c cd\cos\varnothing & -m_c cd\cos\varnothing \\ -m_c cd\sin\varnothing & m_c cd\cos\varnothing & I_c^2 + I_w & -I_c^2 \\ m_c cd\sin\varnothing & -m_c cd\cos\varnothing & -I_c^2 & I_c^2 + I_w \end{vmatrix}$$

$$V(q, \dot{q}) = \begin{bmatrix} 2m_c d\dot{\varnothing}^2\cos\varnothing \\ 2m_c d\dot{\varnothing}^2\sin\varnothing \\ 0 \\ 0 \end{bmatrix}, E(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

In order to eliminate the Lagrange multiplier, we select the null space of $\hat{A}(q)$ as

$$S^T(q) = \begin{bmatrix} cb\cos\varnothing & cb\sin\varnothing & 1 & 0 \\ cb\cos\varnothing & cb\sin\varnothing & 0 & 1 \end{bmatrix}$$

then, equation (3) becomes

$$S^T(q)M(q)(S(q)\dot{\theta} + \dot{S}(q)\theta) + S^T(q)V(q, \dot{q}) = \tau \quad (4)$$

Equation (4) is a type of non-holonomic equation. This type of system cannot be linearized by using the state feedback.

We now present a LPD system model for the mobile robot. Equation (4) becomes

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad (5)$$

where,

$$M_{11} = M_{22} = mc^2b^2 + I_c^2 + I_w$$
$$M_{12} = M_{21} = mc^2b^2 - I_c^2$$
$$N_{11} = m_c cbd(c + \dot{\varnothing}), \ N_{12} = m_c cbd(c - \dot{\varnothing})$$
$$N_{21} = -m_c cbd(c - \dot{\varnothing}), \ N_{22} = -m_c cbd(c + \dot{\varnothing})$$

In equation (5), the variable $\dot{\Phi}$ must be selected as a parameter. Because of the term $\dot{\Phi}^2$, the dynamic equation is not linear with respect to the parameter value $\dot{\Phi}$. After simple algebraic manipulation, we can obtain the LPD system representation of mobile robot system[7]. Define the state variables, input and the output as

$$x_1 \triangleq \theta_1, x_2 \triangleq \theta_2, x_3 \triangleq \dot{\theta}_1, x_4 \triangleq \dot{\theta}_2,$$
$$u = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, y = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

then, the state space representation of mobile robot is

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + A_1(\dot{\Phi}(t))x(t) + B_0 u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{6}$$

where,

$$A_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a_{11}^0 & a_{12}^0 \\ 0 & 0 & a_{21}^0 & a_{22}^0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & a_{11} & a_{12} \\ 0 & 0 & a_{21} & a_{22} \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$a_{11}^0 = \frac{m_c c^2 bd(2mc^2b^2 + I_w)}{4mc^2b^2I_c^2 + 2mc^2b^2I_w + 2I_c^2I_w + I_w^2}$$

$$a_{12}^0 = a_{11}^0, \quad a_{21}^0 = -a_{11}^0, \quad a_{22}^0 = -a_{11}^0$$

$$a_{11}^0 = \frac{m_c cbd(2Ic^2b^2 + I_w)\dot{\Phi}}{4mc^2b^2I_c^2 + 2mc^2b^2I_w + 2I_c^2I_w + I_w^2}$$

$$a_{12} = a_{11}, \quad a_{21} = -a_{11}, \quad a_{22} = -a_{11}$$

$$b_{11(=b_{22})} = \frac{mc^2b^2 + I_c^2 + I_w}{4mc^2b^2I_c^2 + 2mc^2b^2I_w + 2I_c^2I_w + I_w^2}$$

$$b_{12(=b_{21})} = \frac{I_c^2 + mc^2b^2}{4mc^2b^2I_c^2 + 2mc^2b^2I_w + 2I_c^2I_w + I_w^2}$$

In the equation (6), controllability matrix $[A_0, B_0]$ is controllable and $[A_1, B_0]$ is controllable except when the variable $\dot{\Phi}(t) = 0$.

2.2. Takagi-Sugeno Fuzzy Model of wheeled Mobile Robot

The fuzzy model proposed by Tagaki and Sugeno is described by IF-THEN rules which represent local linear input-output relations of a nonlinear system[8]. The main feature of a T-S fuzzy model is to express the local dynamics of each fuzzy rule by a linear system model.

The i-th T-S fuzzy model is[8]

If $z_1(t) = M_{i1}$ and .. and $z_p(t) = M_{ip}$

$$\text{THEN} \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t), \end{cases} \quad i = 1, 2, \cdots, r \tag{7}$$

The final outputs of the fuzzy systems are inferred as follows:

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^r w_i(z(t))\{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^r w_i(z(t))} \\ &= \sum_{i=1}^r h_i(z(t))\{A_i x(t) + B_i u(t)\} \end{aligned} \tag{8.a}$$

$$\begin{aligned} y(t) &= \frac{\sum_{i=1}^r w_i(z(t)) C_i x(t)}{\sum_{i=1}^r w_i(z(t))} \\ &= \sum_{i=1}^r h_i(z(t)) C_i x(t) \end{aligned} \tag{8.b}$$

Note that the elements of matrices A_0, B_0 and C are constant and that the only matrix A_1 depends on the value $\dot{\Phi}(t)$. The fuzzification of the DDWMR is performed on the matrix A_1. The fuzzy model for the DDWMR, described by the equation (7), becomes

If $\dot{\Phi}(t) = M_i$

$$\text{THEN} \begin{cases} \dot{x}(t) = [A_0 + A_{li}]x(t) + B_0u(t) \\ y(t) = Cx(t), \end{cases}$$
$$i = 1, 2, \cdots, r \tag{9}$$

The final outputs of the fuzzy systems are inferred as follows:

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^{r} w_i(z(t))\{[A_0 + A_{1i}]x(t) + B_0u(t)\}}{\sum_{i=1}^{r} w_i(z(t))} \\ &= \sum_{i=1}^{r} h_i(z(t))\{[A_0 + A_{1i}]x(t) + B_0u(t)\} \\ &= A_0x(t) + B_0u(t) + \sum_{i=1}^{r} h_i(z(t))A_{1i}x(t) \\ y(t) &= Cx(t) \end{aligned} \tag{10}$$

The equation (10) is a fuzzy state-space representation of DDWMR.

III. Control of Mobile Robot

We are now state a controller structure presented in this paper, and a new control design algorithm for mobile robot.

3.1. Controller Structure

The most important control strategy of physical systems is reference tracking. To achieve this objective, the control structure is shown in Fig. 2. In Fig. 2, control parameters in the block are all fuzzy controller. And it is shown in Fig. 2 that the controller has two control parameters one of which is state feedback and the other is control gain with integrator. The input signal is described by

$$u(t) = -\left[F(\mu)x(t) + K(\mu)\int e(t)dt\right] \tag{11}$$

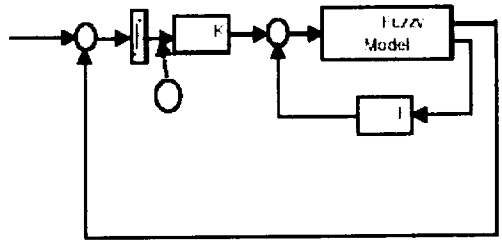

Fig. 2. Controller structure

where, $F(\mu)$ is a fuzzy state feedback gain matrix and $K(\mu)$ is a fuzzy integrator gain matrix. In order for obtain controller gains $F(\mu)$ and $K(\mu)$, it is needed to simplify control input or controller structure. The new state x_{n+1} can be defined at in the Fig. 1. Then the dynamic equation becomes

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_{n+1}(t) \end{bmatrix} = \begin{bmatrix} A_0 \sum_i \mu_i A_i(\mu_i) & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_{n+1}(t) \end{bmatrix} + \begin{bmatrix} B_0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ I \end{bmatrix} r(t) \tag{12.a}$$

$$y(t) = [c \quad 0] \begin{bmatrix} x(t) \\ x_{n+1}(t) \end{bmatrix} \tag{12.b}$$

and, the control input is

$$u(t) = [F(\mu) \quad K(\mu)] \begin{bmatrix} x(t) \\ x_{n-1}(t) \end{bmatrix} \tag{13}$$

It is known by the equation (13) that the control input is state feedback for the system described by the equation (12).

3.2. Regional Pole Placement

The LMI region is defined following definition[6].

Definition 1. LMI regions are convex subset D of the complex plan characterized by

$$D = \{z \in C : L + Mz + M^T z^*\} \tag{14}$$

where M and L are fixed real matrices, and z and z* are complex valued scalar and its complex conjugate pair. The matrix valued function

$$f_D(z) = \{L + Mz + M^T z^*\} \tag{15}$$

is called the characteristic function of the region D. We are now state a local pole placement. Let M_i be the fuzzy model obtained by substituting the $\dot{\Phi}$ as i-th sample. And select a function, $\mu(p^{i,j,\cdots,l})$, a local convex function, then the following theorem states the algorithm of obtaining the controller gain matrix for M_i.

Theorem 1: For M_i, the closed loop poles lie in the LMI region D, described by equation (14), where

$$L = L^T = [\lambda_{jk}]_{1 \le j,k \le m}, \quad M = [m_{jk}]_{1 \le j,k \le m}$$

if and only if there exists a symmetric matrix X satisfying following four inequalities.

$$\begin{aligned} &[\lambda_{jk} X + m_{jk} A_i^{CL} X + m_{kj} A_i^{CL^T}]_{1 \le j,k \le m} < 0 \\ &X > 0 \end{aligned} \tag{16}$$

where A_i^{CL} is

$$A_i^{CL} = \begin{bmatrix} A_0 + \sum_i \mu_i A_1(\mu_1) + B_0 \sum_i \mu_i F(\mu) & B_0 \sum_i \mu_i K(\mu) \\ -C & 0 \end{bmatrix}$$

Proof) Proof of this theorem is simple extension Chilali and Gahinet's work [6] (QED).

Theorem 1 states the local regional pole placement of the M_i. Because the equation (16) is not convex, we cannot obtain the controller gain matrix. Define a matrix $Y_i := F_i X$, then conditions of local pole placement is summarized by Theorem 2.

Theorem 2: The closed loop poles lie in the LMI region D if and only if there exists a symmetric matrix X satisfying following inequalities.

$$\begin{aligned} &\left| \begin{matrix} \lambda_{jk} X + m_{jk} \left(\begin{bmatrix} A_0 + \sum_i \mu_i A_i(\mu_i) & 0 \\ -C & 0 \end{bmatrix} X + \begin{bmatrix} B_0 \\ 0 \end{bmatrix} Y_i \right) \\ + m_{kj} \left(\begin{bmatrix} A_0 + \sum_i \mu_i A_i(\mu_i) & 0 \\ -C & 0 \end{bmatrix} X + \begin{bmatrix} B_0 \\ 0 \end{bmatrix} Y_i \right)^T \end{matrix} \right|_{1 \le j,k \le m} < 0 \\ &X > 0 \end{aligned} \tag{17}$$

the i-th state-feedback gain matrix is

$$[F_i \ K_i] = Y_i K^{-1} \tag{18}$$

Proof) The proof of this theorem is very simple extension of the results of Chilali and Gahinet's work [6] (QED).

The theorem 1 and theorem 2 shows the local regional pole-placement condition and the way of finding local controller gains. The global pole-placement condition and global controller gain can be achieved by using approximated plant. In order for global pole-placement, the control input, made up of local controller gain, is selected by

$$u(t) = -\left[\sum_i \mu_i(\dot{\Phi}) F_i(\mu) x(t) + \sum_i \mu_i(\dot{\Phi}) K_i(\mu) \int e(t) dt \right] \tag{19}$$

By noting the equation (19), the controller gain is made up of local controller gains and which is convex combination of local controller gains between [(i,j, l)~(i-1,j, l)]. The following theorem states the global regional pole-placement.

Theorem 3: Assume that the plant model is modeled by the equation (10) and local controller gains are obtained by the equation (18) for local fuzzy model. Then the closed loop poles are lie in the desired region.

Proof). The proof of this theorem is very simple extension of the results of Chilali and Gahinet's work (QED).

The theorem 3 states the global global-placement condition and controller design procedure is summarized as 1) sampling model 2) design local controller 3) combine it.

IV. SIMULATION

In simulation, the robot considered is MIROSOT soccer robot, and detailed specifications are summarized in the table 1. The mass of the robot is 0.0612 Kg m/sec_2 and the mass of wheels is 0.0051 kg m/sec_2. And other parametersused in this paper were

$$b=35mm, \quad c=r/2b, \quad d=10mm$$

The robot inertia except wheels and rotor is 0.052 sec *Kg cm* and motor rotor inertia for wheels and wheel axis is 0.0176 2 sec *Kg cm*. These parameters were actually measured and computed for MIROSOT robot designed Yujin Robotics corp. In this paper, the maximum velocity of the wheel was the maximum velocity of the motor specification.

By using parameters described above, state space matrices for the mobile robot are

$$A_0=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.0816 & 0.0816 \\ 0 & 0 & -0.0816 & -0.0816 \end{bmatrix}, \quad A_1=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0333\dot{\phi}_r & -0.0333\dot{\phi}_r \\ 0 & 0 & 0.0333\dot{\phi}_r & -0.0333\dot{\phi}_r \end{bmatrix},$$

Table 1. The specifications of MIROSOT robot.

Size	70x70x70 mm
Wheel diameter	22.5 mm
Rpm	8000
Gear ratio	8:1

$$B_0=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 25.0222 & -19.2256 \\ -19.2256 & 25.0222 \end{bmatrix}, \quad C=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Membership functions of this paper are shown in the Fig. 3. Fig. 4 and Fig. 5 are simulation results for pulse reference input.

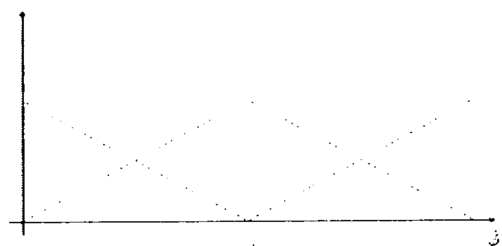

Fig. 3. The membership functions

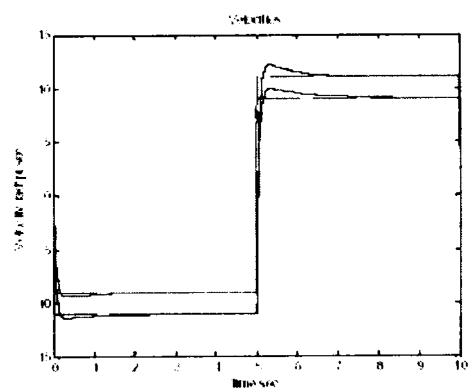

Fig. 4. Velocities

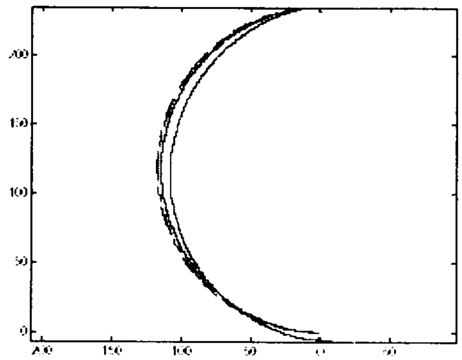

Fig. 5. Tracking result

The controller is the PI control loop which is designed well known LQ algorithm. In order to the diagonal gain matrix, some technical trick is used for selecting weighting matrices. It is shown by Fig. 4 that the velocity following error becomes zero. But the overshoot is occurred and by it, the tracking results include error.

V. CONCLUSION

In this paper, the DDWMR is considered. The T-S fuzzy model for DDWMR is presented and control algorithm is suggested. The controller is the PI control loop which is designed well known LQ algorithm. In order to the diagonal gain matrix, some technical trick is used for selecting weighting matrices. It is shown by this paper that the presented algorithm is more easy way of control of the DDWMR and that the result of this paper can be applicable to car-like WMR.

REFERENCE

1) S.H. Kim, J.S. Choi, J.K. Kim and B.K. Kim, 1997, A Cooperative Strategy for Robots, Symposium of Control Measurement, Automation and Robotics., pp. 283-287.

2) X. Wang and M. Sugisaka,1999, Learning Robot Using Genetic Programming, *Proc. of the 14th KACC.*, E- 292-297.

3) J.M. Yang and J.H. Kim, 1997, Tracking Control of Wheeled Mobile Robots Using Variable Structure Systems, *Proc. of the 2nd Asian Control Conference, Seoul, Korea,* Jul. 22-25, I-381-384.

4) J.Sh. Kang, Ch.W. Rhim, 2003, LPD System Modeling Sysem Modeling and Control of Mobile Robot, Intl. J. Contr. Auto, Sys. Vol 1, No. 2,.

5) K. Tanaka and H.O. Wang, 2001, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach,* Wiley-Interscience Publication, pp 5-37,.

6) Chilali, M. and P. Gahinet, 1995, H∞ Design with Pole-Placement Constraints: an LMI Approach, IEEE Trans. Auto. Contr., Vol. 41, No. 3.,