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Heat Transfer Correlation in Horizontal Fluid Layer Heated Uniformly from Below

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밑면이 일정 열속으로 가열되는 수평 유체층에서의 열전달 상관관계

김 민 찬* · 이 상 백*

ABSTRACT

The buoyancy effects in horizontal fluid layer heated uniformly from below was analysed. Based on the stability criteria and the boundary-layer instability model, a new heat transfer correlation which can cover whole range of Rayleigh number was derived. Our theoretical results predicted the experimental results, quite reasonably.

Key words : Buoyancy effect, stability analysis, heat transfer correlation.

Ⅰ. Introduction

When an initially quiescent fluid layer is heated from below with a certain Rayleigh number exceeding critical value, the buoyancy-driven convective motion occurs. This convective motion driven by buoyancy forces has attracted many researcher's attention from the beginning of this century. It is well-known that buoyancy-driven convection plays an important roles in many engineering problem, such as chemical vapor deposition, solidification, electroplating and also many other conventional processes involving heat and mass transfer.

The important problem in buoyancy-driven convection will be its heat transfer characteristics in the thermally fully-developed state. For analysing this problem Howard[1)] proposed boundary-layer instability model that for very high Rayleigh number case the heat transfer characteristics have close relationship with stability criteria. Busse[2)] modified Howard's concept by considering the heat transfer resistance of upper boundary. Long[3)], Cheung[4)] and Arpaci[5)] derived backbone equation to predict the heat transfer in horizontal fluid layer. By incorporating their stability criteria into the boundary-layer instability model, Choi et al. have derived new heat transfer correlations for horizontal

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fluid layer[6] fluid saturated porous layer[7], plane Couette flow[8], and plane Poiseuille flow[9]. Their resulting heat transfer correlations are in good agreement with a great deal of available experimental data.

In this study we consider the buoyancy effects in horizontal fluid layer heated from below. The new heat transfer correlation is proposed and compared with experimental data.

II. Turbulent Heat Transport

The possibility of connecting stability criteria to the heat transport on the turbulent thermal convection was investigated by Howard[1]. He postulated that at a large Rayleigh number the convective instability, in the form of thermal, sets in after a time t_*, and the thermal break-up the boundary layer after a time much shorter than t_*. It was further assumed that after the break-up of boundary layer the system is restored to quiescent state. According to the Howard's concept, we assume that the onset of thermal can be described by the above stability analysis, and that the turbulent heat transfer would be governed by the narrow boundary layer like a conduction film of thickness δ_* near the heated surface. δ_* is usually called the conduction layer thickness.

Busse[2] modified the Howard's concept such that the heat transport resistances exist near the upper boundary as well as the lower one. This so-called boundary layer instability model is schematized in Fig. 1. According to this model the Nusselt number in the fully-developed turbulent state is expressed as:

$$\mathrm{Nu} = \frac{1}{2}\frac{\delta_*}{\mathrm{d}} \quad \text{for} \quad Ra \rightarrow \infty \tag{1}$$

Long[3], Cheung[4], and Arpaci[5] analysed the buoyancy-driven turbulent heat transport semi-theoretically and showed that the heat transport characteristics for $Ra \rightarrow \infty$ would be independent of the fluid-layer depth, like the Howard's and Busse's concept. By slight modification of their model, the following heat transfer correlation for the present system can be obtained:

$$Nu = \frac{ARa_q^{1/4}}{1 - BRa_q^{-1/12}} \tag{2}$$

where A and B are the undetermined constants.

By transforming eq. (1) the heat transport in the fully-developed turbulent state may be expressed as:

$$Nu = \frac{1}{2}\left(\frac{Ra}{2Ra_\delta}\right)^{1/3} \quad \text{for} \quad Ra \rightarrow \infty \tag{3}$$

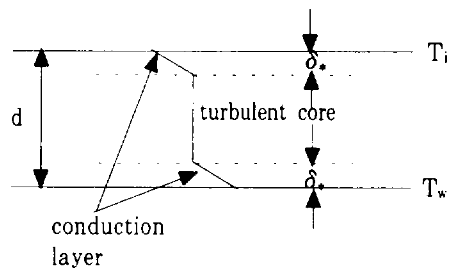

Fig. 1. Turbulent heat transport model.

where $Ra_\delta = g\beta\Delta T_\delta \delta_*^3/(\alpha\nu)$ is the Rayleigh number based on the conduction thickness δ_* and the temperature difference over the conduction layer thickness ΔT_δ. Following the boundary-layer instability model of Fig. 1, ΔT_δ is the half of the total temperature difference ΔT. By using the relation of $Ra_q = Ra \cdot Nu$, eq. (3) can be replaced by

$$Nu = \frac{1}{2}\left(\frac{Ra_q}{Ra_\delta}\right)^{1/4} \quad \text{for} \quad Ra_q \rightarrow \infty \tag{4}$$

Following the Howard's concept δ_* may be replaced by $\delta_{T,c}$. $\delta_{T,c}$ is thermal penetration depth at the onset condition of buoyancy-driven convection. From the result of Kim and Hyun[10], and the relations of $Ra^{*}=\frac{g\beta q_{u}d^{4}}{k\alpha\nu}\tau^{2}$, $\delta_{T,c}=3.21\tau_{c}^{1/2}$, and $\frac{k\Delta T}{q_{u}d}=\frac{2}{\sqrt{\pi}}\tau_{c}^{1/2}$, $Ra_{\delta_{T,c}}$ can be obtained as:

$$Ra_{\delta_{T,c}}=747.27 \tag{5}$$

By combining eq. (4) and (5) the resulting heat transfer correlation for the fully-developed turbulent state is expressed as follows:

$$Nu=0.0956Ra_{q}^{1/4} \quad \text{for} \quad Ra_{q}\rightarrow\infty \tag{6}$$

Ⅲ. Heat Transfer Correlation

The finite-amplitude heat transfer characteristics slightly over $Ra_{q,c}(=1296)$ can be obtained by using the shape assumption of Stuart[11]. For the region of $Ra_{q}\rightarrow Ra_{q,c}$, the Nusselt number can be expressed as:

$$\frac{1}{Nu}=1-\frac{\Gamma}{Ra_{q}}(Ra_{q}-Ra_{q,c}) \quad \text{for} \quad Ra_{q}\rightarrow Ra_{q,c} \tag{7}$$

The constant Γ is obtained from the distribution of disturbance quantities at $Ra_{q}=Ra_{q,c}$:

$$\Gamma=\frac{\left(\int_{0}^{1}w_{1}\theta_{1}dz\right)^{2}}{\int_{0}^{1}(w_{1}\theta_{1})^{2}dz}=0.6313 \tag{8}$$

Assembling the eqs. (2), (6) and (7), for the whole range of Ra_{q}, we can derive a new heat transfer correlation of the present system of large Prandtl number fluid as

$$Nu=1+\frac{0.0956(Ra_{q}^{1/4}-1296^{1/4})}{1-1.404Ra_{q}^{-1/4}} \tag{9}$$

The above prediction agree fairly well with the experimental results of Nielsen and Sabersky[12], as shown in Fig. 2. It is noted that $Nu\equiv1$ for $Ra_{q}=1296$. This value corresponds to that of conduction state.

Ⅳ. Conclusion

The new heat transfer correlation is derived based on the boundary layer instability model. It is interesting that our theoretical predictions have close agreement with experimental results. Therefore, it may be stated that our propagation theory is a powerful tools to examine the buoyancy-driven phenomena in horizontal fluid layers.

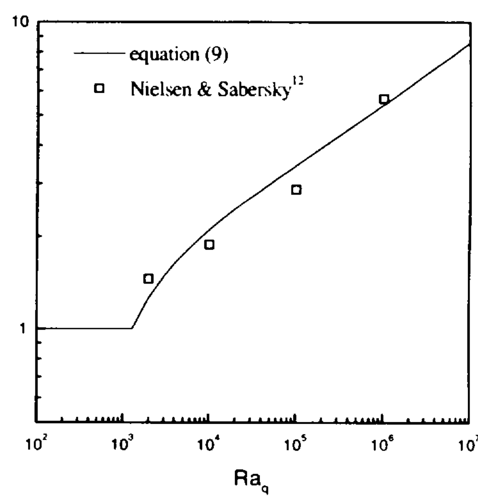

Fig. 2. Comparison with Nu vs. Ra_q for large Prandtl number.

요 약

밑면이 일정 열속으로 가열되는 수평 유체층에서

부력 효과를 해석하였다. 안정성 해석 결과 및 경계층 불안정성 모델을 근거로 Rayleigh 수 전범위에 걸친 열전달 상관식을 유도하였다. 본 연구의 해석결과는 기존의 실험결과를 잘 설명하여 준다.

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